Model Exam 1

MATHEMATICS (Geometry)

30 Marks

Answer the following questions :

1 Choose the correct answer from those given :

1 The slope of the straight line $3\,x + 2\,y = 1$ is ....

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$

2 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm.
, then $MN \in$ ....

(a) $]8 , \infty[$ (b) $]3 , 5[$ (c) $]0 , 2[$ (d) $]2 , 8[$

3 The measurement of any angle of the regular hexagon is ....

(a) 90° (b) 108° (c) 120° (d) 135°

4 ABCD is a cyclic quadrilateral , m ($\angle$ A) = 70° , then m ($\angle$ C) equals ....

(a) 25° (b) 20° (c) 110° (d) 100°

5 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then $\angle$ B is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

6 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 130° (b) 90° (c) 50° (d) 180°

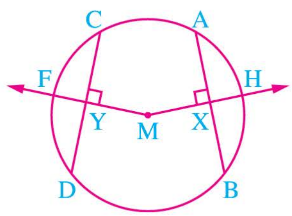
---

2 [a] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{CD}$

Prove that : HX = FY

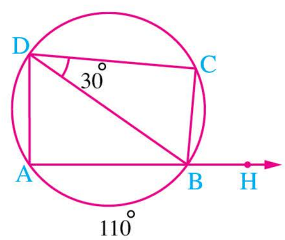


[b] In the opposite figure :

$H \in \overrightarrow{AB}$, m $(\overset{\frown}{AB})$ = 110°

, m ($\angle$ CDB) = 30°

Find : m ($\angle$ HBC)



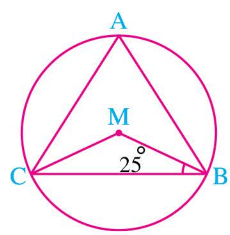
---

3 [a] In the opposite figure :

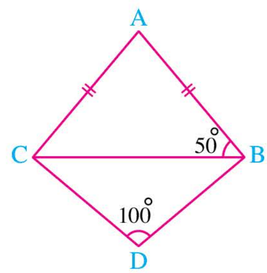
ABC is a triangle drawn in the circle M

, m ($\angle$ MBC) = 25°

Find : m ($\angle$ BAC)



[b] In the opposite figure :



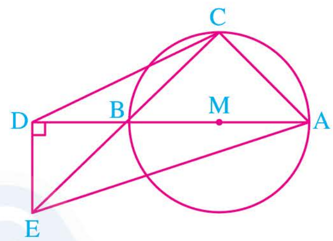
AB = AC , m (∠ D) = 100°

, m (∠ ABC) = 50°

Prove that : ABDC is a cyclic quadrilateral.

---

4 [a] In the opposite figure :



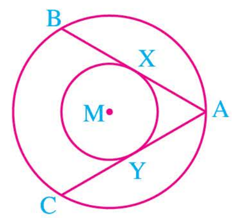
$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral

[b] In the opposite figure :



Two concentric circles of centre M

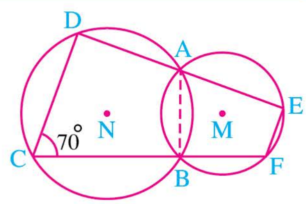
, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle

and tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

---

5 [a] In the opposite figure :



M and N are two intersecting circles at A and B

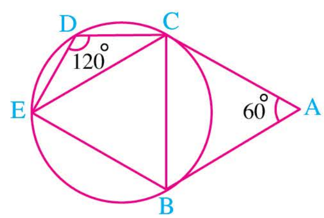
, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and

the circle N at D , $\overleftrightarrow{AB}$ is drawn to intersect the circle M at

F and the circle N at C , m (∠ BCD) = 70°

1 **Find :** m (∠ EFB)   2 **Prove that :** $\overline{CD}$ // $\overline{EF}$

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C

, m (∠ BAC) = 60° , m (∠ CDE) = 120°

Prove that : 1 Δ BCE is an equilateral triangle.

2 $\overline{AC}$ // $\overline{BE}$

Answer the following questions :

1 **Choose the correct answer from those given :**

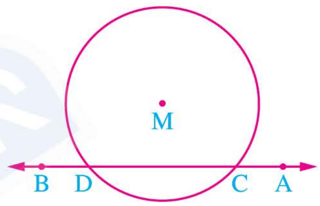
1. $\angle$ A and $\angle$ B are two complementary angles , $\angle$ B and $\angle$ C are two supplementary angles , m ($\angle$ A) = 30° , then m ($\angle$ C) = ...°

   (a) 30 (b) 60 (c) 90 (d) 120

2. If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$ and the radius length of one of them equals 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

   (a) 5 (b) 6 (c) 11 (d) 16

3. **In the opposite figure :**



   $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

   (a) $\{C , D\}$ (b) $\overline{CD}$

   (c) $\overleftrightarrow{CD}$ (d) $\varnothing$

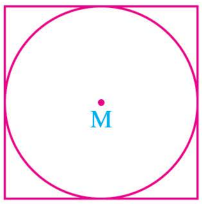
4. A circle can be drawn passing through the vertices of a ....

   (a) rhombus. (b) parallelogram. (c) trapezium. (d) rectangle.

5. The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals .... cm.

   (a) 6 (b) 8 (c) 10 (d) 20

6. **In the opposite figure :**



   If the side length of the square = 10 cm.

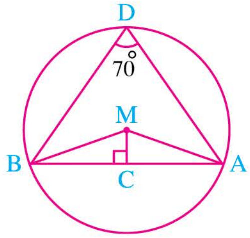
   , then the surface area of the circle = ... cm^2

   (a) 100 π (b) 25 π

   (c) 50 π (d) 40 π

---

2 **[a] In the opposite figure :**

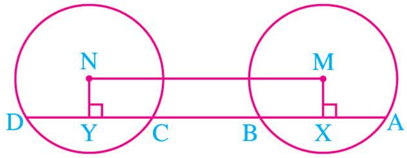


$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, m ($\angle$ ADB) = 70°

Find : m ($\angle$ AMC)

[b] In the opposite figure :



M and N are two congruent circles

, AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.

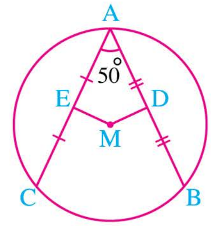
3 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M , D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ and m ($\angle$ BAC) = 50°

Find : m ($\angle$ DME)



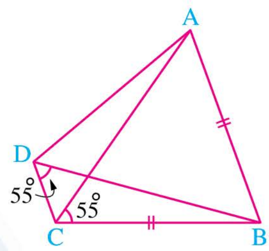
[b] **In the opposite figure :**

AB = BC

, m ($\angle$ ACB) = 55°

and m ($\angle$ BDC) = 55°

Prove that : The figure ABCD is a cyclic quadrilateral.



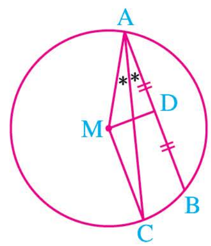
4 [a] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overrightarrow{AC}$ bisects $\angle$ BAM and intersects the circle M at C

If D is the midpoint of $\overline{AB}$

, **prove that :** $\overline{DM} \perp \overline{CM}$



[b] $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y respectively and intersects $\overleftrightarrow{BD}$ at E **Prove that :** CX = YE

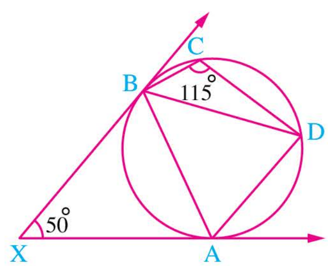
5 [a] **In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m ($\angle$ AXB) = 50° , m ($\angle$ DCB) = 115°

Prove that : 1 $\overrightarrow{AB}$ bisects $\angle$ DAX

2 BD = BA

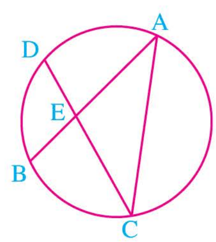


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



Answer the following questions :

1 Choose the correct answer from those given :

1. The measure of the inscribed angle is .... the measure of the central angle subtended by the same arc.

   (a) half (b) twice (c) quarter (d) third

2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

3. Two distant circles M and N with radii lengths 6 cm. and 8 cm. respectively , then MN .... 14 cm.

   (a) < (b) > (c) = (d) ≤

4. The angle of measure 40° is the complemented angle of the angle of measure ....°

   (a) 320 (b) 140 (c) 60 (d) 50

5. The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm^2.

   (a) 2 (b) 14 (c) 24 (d) 48

6. In the cyclic quadrilateral ABCD , if m (∠ A) = $\frac{1}{2}$ m (∠ C) , then m (∠ A) = ...°

   (a) 20 (b) 30 (c) 60 (d) 120

2 [a] In the opposite figure :

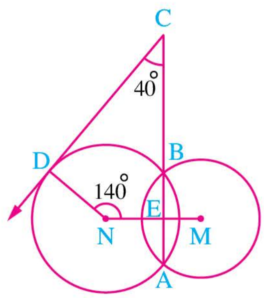
M and N are two intersecting circles at A and B

, C ∈ $\overrightarrow{AB}$, $\overline{AC} \cap \overline{MN}$ = {E}

, D ∈ the circle N , m (∠ DNM) = 140°

and m (∠ C) = 40°

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



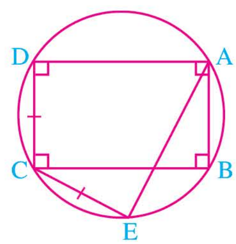
[b] In the opposite figure :

ABCD is a rectangle inscribed in a circle

, the chord $\overline{CE}$ is drawn

where CE = CD

Prove that : AE = BC



3 [a] State two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**

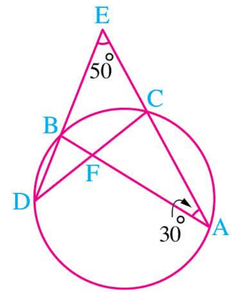
$\overline{AB} \cap \overline{CD} = \{F\}$, $\overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

, m ($\angle$ A) = 30°

, m ($\angle$ E) = 50°

Find : $\boxed{1}$ m ($\overset{\frown}{AD}$)

$\boxed{2}$ m ($\angle$ AFD)



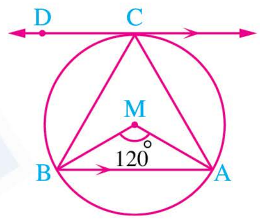
---

4 [a] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m ($\angle$ AMB) = 120°

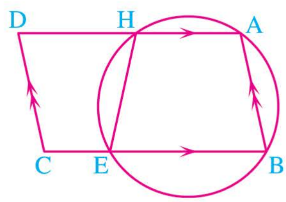
Prove that : The triangle CAB is an equilateral triangle.



[b] **In the opposite figure :**

ABCD is a parallelogram.

Prove that : HDCE is a cyclic quadrilateral.



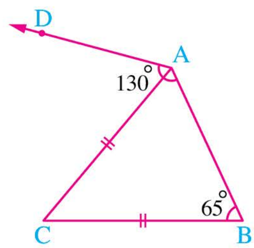
---

5 [a] **In the opposite figure :**

AC = BC , m ($\angle$ ABC) = 65°

, m ($\angle$ DAB) = 130°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



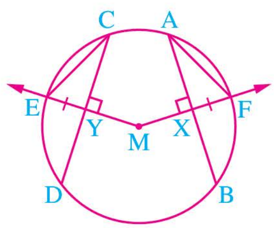
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : $\boxed{1}$ AB = CD $\boxed{2}$ AF = CE



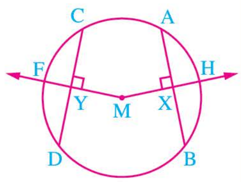
Answers of model 1

1

1 b　　2 d　　3 c

4 c　　5 a　　6 b

2



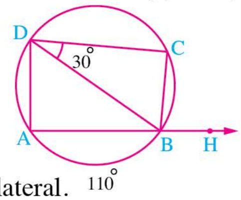
[a] $\because$ AB = CD , $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{CD}$

$\therefore$ MX = MY

, $\because$ MH = MF = r

$\therefore$ HX = FY (Q.E.D.)



[b] $\because$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$)

= $\frac{1}{2}$ × 110°

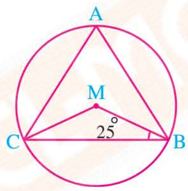
= 55°

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ HBC) = m ($\angle$ CDB) + m ($\angle$ ADB)

= 30° + 55° = 85° (The req.)

3



[a] In Δ BMC :

$\because$ MB = MC = r

$\therefore$ m ($\angle$ MCB)

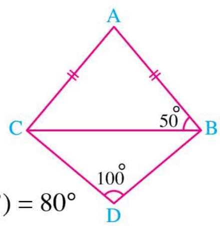
= m ($\angle$ MBC) = 25°

$\therefore$ m ($\angle$ BMC) = 180° – (25° + 25°) = 130°

, $\because$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2}$ × 130° = 65° (The req.)



[b] In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC)

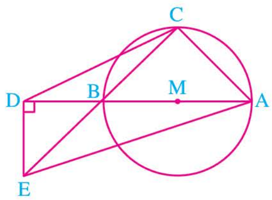
= 50°

$\therefore$ m ($\angle$ A) = 180° – (50° + 50°) = 80°

, $\because$ m ($\angle$ A) + m ($\angle$ D) = 80° + 100° = 180°

$\therefore$ ABDC is a cyclic quadrilateral. (Q.E.D.)

4



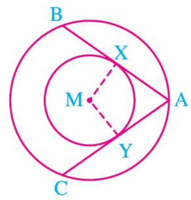
[a] $\because$ $\overline{AB}$ is a diameter of the circle.

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ ACE) = m ($\angle$ ADE)

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Q.E.D.)



[b] **Construction :**

Draw $\overline{MX}$, $\overline{MY}$

Proof :

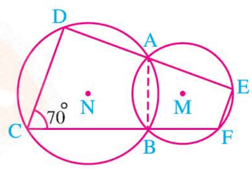
$\because$ $\overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle at X , Y respectively

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radius length of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5



[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD)

= 180° – 70° = 110°

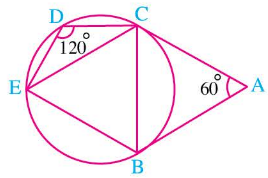
, $\because$ ABFE is a cyclic quadrilateral and $\angle$ BAD is exterior of it.

$\therefore$ m ($\angle$ EFB) = m ($\angle$ BAD) = 110° (First req.)

$\therefore$ m ($\angle$ EFB) + m ($\angle$ BCD) = 110° + 70° = 180°

and they are interior angles in the same side of $\overleftrightarrow{FC}$

$\therefore$ $\overline{CD}$ // $\overline{EF}$ (Second req.)



[b] $\because$ $\overline{AB}$, $\overline{AC}$ are tangent-segments to the circle

$\therefore$ AB = AC

$\therefore$ m ($\angle$ ACB) = $\frac{180° - 60°}{2}$ = 60° (1)

$\therefore$ m ($\angle$ BEC) (inscribed)

= m ($\angle$ ACB) (tangency) = 60° (2)

, $\because$ EBCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ EBC) = 180° – 120° = 60° (3)

$\therefore$ From (2) , (3) in Δ EBC :

$\therefore$ m ($\angle$ BCE) = 60°

$\therefore$ Δ BCE is equilateral. (Q.E.D. 1)

From (1) , (3) : $\therefore$ m ($\angle$ ACB) = m ($\angle$ EBC) and they are alternate angles

$\therefore$ $\overline{AC}$ // $\overline{BE}$ (Q.E.D. 2)

Answers of model 2

1

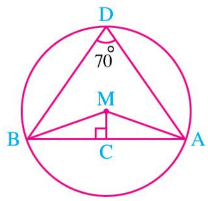
1 d	2 a	3 b
4 d	5 c	6 b

2

[a] $\because$ m ($\angle$ AMB) = 2 m ($\angle$ ADB)

= 2 × 70° = 140°

(central and inscribed angles subtended by $\overset{\frown}{AB}$)



In Δ ABM : $\because$ $\overline{MC} \perp \overline{AB}$

, MA = MB = r

$\therefore$ $\overrightarrow{MC}$ bisects $\angle$ AMB

$\therefore$ m ($\angle$ AMC) = $\frac{1}{2}$ m ($\angle$ AMB) = $\frac{1}{2}$ × 140° = 70°

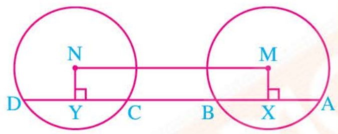
(The req.)

[b] $\because$ M , N are two congruent circles

, AB = CD

, $\overline{MX} \perp \overline{AB}$

, $\overline{NY} \perp \overline{CD}$



$\therefore$ MX = NY , $\overline{MX}$ // $\overline{NY}$

$\therefore$ MXYN is a rectangle. (Q.E.D.)

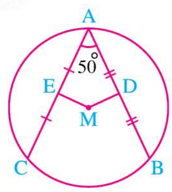
3

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$

$\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{AC}$



$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ AEM) = 90°

From the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° – (90° + 90° + 50°) = 130°

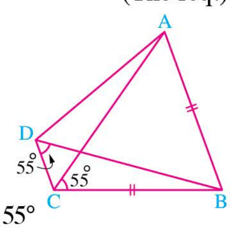
(The req.)

[b] In Δ ABC :

$\because$ AB = BC

$\therefore$ m ($\angle$ BAC)

= m ($\angle$ ACB) = 55°



, $\because$ m ($\angle$ BDC) = m ($\angle$ BAC) = 55°

and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] In Δ AMC :

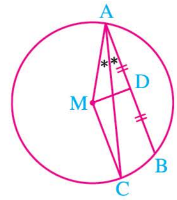
$\because$ AM = MC = r

$\therefore$ m ($\angle$ MAC) = m ($\angle$ ACM)

, $\because$ m ($\angle$ BAC) = m ($\angle$ MAC)

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACM)

and they are alternate angles.



$\therefore$ $\overline{AB}$ // $\overline{CM}$

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$, $\because$ $\overline{AB}$ // $\overline{CM}$

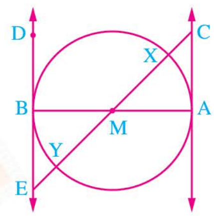
$\therefore$ $\overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] $\because$ $\overleftrightarrow{AC}$ is a tangent to the circle M at A

$\therefore$ $\overline{MA} \perp \overleftrightarrow{AC}$

$\therefore$ m ($\angle$ CAM) = 90°

, $\because$ $\overleftrightarrow{BD}$ is a tangent to the circle M at B



$\therefore$ $\overline{MB} \perp \overleftrightarrow{BD}$ $\therefore$ m ($\angle$ EBM) = 90°

$\therefore$ In ΔΔ CAM , EBM :

$\begin{cases} m (\angle CAM) = m (\angle EBM) = 90° \\ m (\angle AMC) = m (\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$

$\therefore$ The two triangles are congruent and we deduce that CM = EM

, $\because$ XM = YM (lengths of two radii)

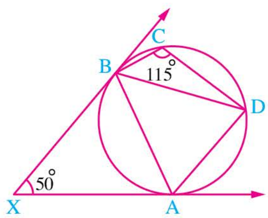
, by subtracting

$\therefore$ CX = YE (Q.E.D.)

5

[a] $\because$ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB



$\therefore$ In Δ ABX

m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DCB) = 180°

$\therefore$ m ($\angle$ BAD) = 180° – 115° = 65°

$\therefore$ m ($\angle$ XAB) = m ($\angle$ BAD)

$\therefore$ $\overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.1)

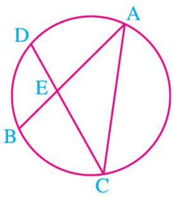
$\because$ m ($\angle$ ADB) (inscribed)

= m ($\angle$ XAB) (tangency) = 65°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ ADB)

$\therefore$ In Δ ABD : BD = BA (Q.E.D.2)

[b] $\because$ AB = CD



$\therefore$ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$)

Subtracting m ($\overset{\frown}{BD}$) from both sides

$\therefore$ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{BC}$)

$\therefore$ m ($\angle$ ACD) = m ($\angle$ BAC)

$\therefore$ In Δ ACE : AE = CE

$\therefore$ Δ ACE is an isosceles triangle. (Q.E.D.)

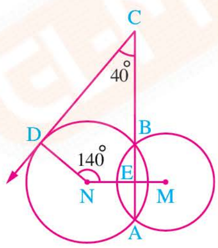
Answers of model 3

1

1 a	2 a	3 b
4 d	5 c	6 c

2

[a] $\because$ $\overleftrightarrow{MN}$ is the line of centres



, $\overline{AB}$ is the common chord.

$\therefore$ $\overline{AB} \perp \overline{MN}$

$\therefore$ m ($\angle$ BEN) = 90°

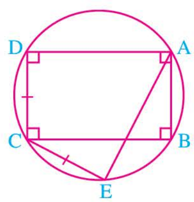
In the quadrilateral CDNE :

$\therefore$ m ($\angle$ CDN) = 360° – (140° + 40° + 90°) = 90°

$\therefore$ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

$\therefore$ $\overrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because$ AB = CD



(properties of the rectangle)

, $\because$ CE = CD

$\therefore$ AB = CE

$\therefore$ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CE}$) and adding m ($\overset{\frown}{BE}$) to both sides.

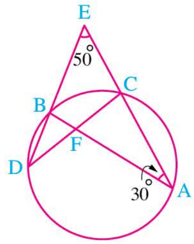
$\therefore$ m ($\overset{\frown}{AE}$) = m ($\overset{\frown}{BC}$)

$\therefore$ AE = BC (Q.E.D.)

3

[a] State by yourself.

[b] $\because$ m ($\overset{\frown}{BC}$) = 2 m ($\angle$ A)



= 2 × 30° = 60°

, $\because$ m ($\angle$ E)

$= \frac{1}{2}$ [m ($\overset{\frown}{AD}$) – m ($\overset{\frown}{BC}$)]

$\therefore$ 50° = $\frac{1}{2}$ [m ($\overset{\frown}{AD}$) – 60°]

$\therefore$ 100° = m ($\overset{\frown}{AD}$) – 60°

$\therefore$ m ($\overset{\frown}{AD}$) = 160° (First req.)

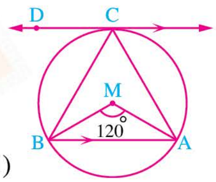
, $\because$ m ($\angle$ AFD) = $\frac{1}{2}$ [m ($\overset{\frown}{AD}$) + m ($\overset{\frown}{BC}$)]

$\therefore$ m ($\angle$ AFD) = $\frac{1}{2}$ [160° + 60°] = 110°

(Second req.)

4

[a] $\because$ m ($\angle$ ACB)



$= \frac{1}{2}$ m ($\angle$ AMB) = 60°

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$) (1)

$\because$ $\overleftrightarrow{CD}$ // $\overline{AB}$

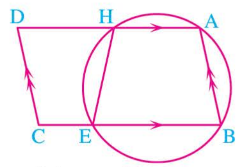
$\therefore$ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore$ Δ CAB is equilateral. (Q.E.D.)

[b] $\because$ $\overline{AB}$ // $\overline{DC}$, $\overleftrightarrow{AD}$ is a transversal to them.



$\therefore$ m ($\angle$ A) + m ($\angle$ D) = 180° (1)

but $\angle$ CEH is an exterior angle of the cyclic quadrilateral ABEH

$\therefore$ m ($\angle$ CEH) = m ($\angle$ A) (2)

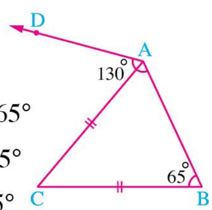
From (1) and (2) :

$\therefore$ m ($\angle$ CEH) + m ($\angle$ D) = 180°

$\therefore$ HDCE is a cyclic quadrilateral. (Q.E.D.)

5

[a] In Δ ABC :



$\because$ AC = BC

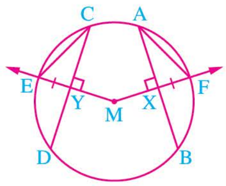
$\therefore$ m ($\angle$ BAC) = m ($\angle$ ABC) = 65°

$\therefore$ m ($\angle$ CAD) = 130° – 65° = 65°

, $\because$ m ($\angle$ B) = m ($\angle$ CAD) = 65°

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] $\because$ MF = ME (lengths of two radii)



, XF = YE

$\therefore$ MX = MY

$\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore$ AB = CD (Q.E.D.1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$, $\because \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$, $\because$ AB = CD

$\therefore$ AX = CY

$\therefore$ In $\Delta\Delta$ AXF , CYE

$$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$$

$\therefore \Delta AXF \equiv \Delta CYE$ $\therefore$ AF = CE (Q.E.D.2)

Model Examinations of the School Book on Geometry

Model 1

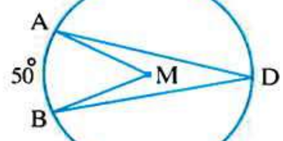
Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

2 **In the opposite figure :**



Circle of centre M

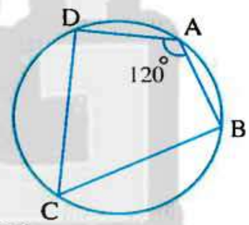
If m $(\widehat{AB}) = 50°$, then m (∠ ADB) = ...

(a) 25° (b) 50° (c) 100° (d) 150°

3 The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

4 **In the opposite figure :**



If m (∠ A) = 120° , then m (∠ C) = ...

(a) 60° (b) 90°

(c) 120° (d) 180°

5 If the straight line L is a tangent to the circle M of diameter length 8 cm.
, then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

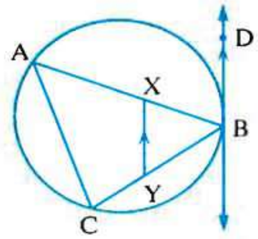
6 The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

2 [a] Complete and prove that :

In a cyclic quadrilateral , each two opposite angles are ....

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$ where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**

Two circles are touching internally at B

, $\overrightarrow{AB}$ is a common tangent

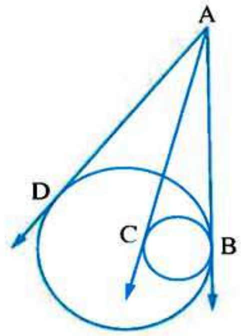
, $\overrightarrow{AC}$ is a tangent to the smaller circle at C

, $\overrightarrow{AD}$ is a tangent to the greater circle at D

, AC = 15 cm. , AB = $(2x - 3)$ cm.

and AD = $(y - 2)$ cm.

Find : The value of each of x and y



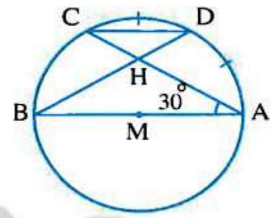
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, C $\in$ the circle M , m ($\angle$ CAB) = 30°

, D is midpoint of $\widehat{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

1 **Find :** m ($\angle$ BDC) and m $(\widehat{AD})$ **2** **Prove that :** $\overline{AB}$ // $\overline{DC}$



4 **[a] In the opposite figure :**

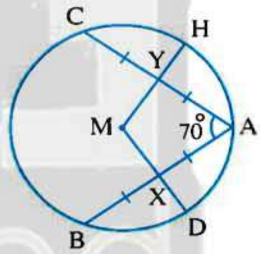
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in circle M

, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

, m ($\angle$ CAB) = 70°

1 **Calculate :** m ($\angle$ DMH)

2 **Prove that :** XD = YH



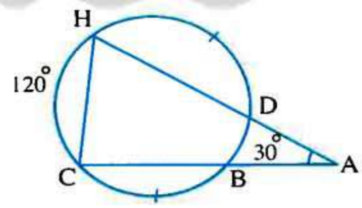
[b] In the opposite figure :

m ($\angle$ A) = 30° , m $(\widehat{HC})$ = 120°

, m $(\widehat{BC})$ = m $(\widehat{DH})$

1 **Find :** m ($\widehat{BD}$ the minor)

2 **Prove that :** AB = AD



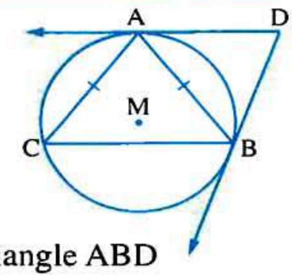
5 **[a] In the opposite figure :**

$\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents of the circle M

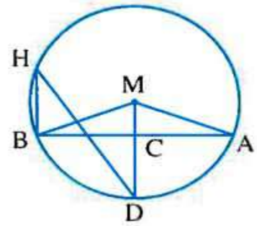
and AB = AC

Prove that :

$\overline{AC}$ is a tangent to the circle passing through the vertices of the triangle ABD



[b] In the opposite figure :



C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D}

, m (∠ MAB) = 20°

Find : m (∠ BHD) and m ($\widehat{ADB}$)

Model 2

1 Choose the correct answer from those given :

1 The measure of the arc which equals half the measure of the circle equals ....

(a) 360° (b) 180° (c) 120° (d) 90°

2 The number of common tangents of two touching circles externally equals ....

(a) 0 (b) 1 (c) 2 (d) 3

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 80°

4 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

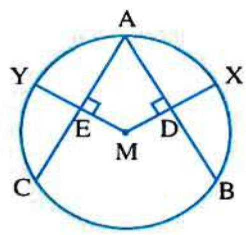
5 ABCD is a cyclic quadrilateral, m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

6 If M , N are two touching circles internally, their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :



AB = AC , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : XD = YE

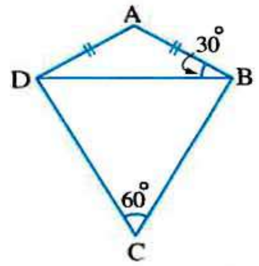
[b] In the opposite figure :

ABCD is a quadrilateral in which AB = AD ,

m (∠ ABD) = 30° ,

m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



3 [a] State two cases of a cyclic quadrilateral.

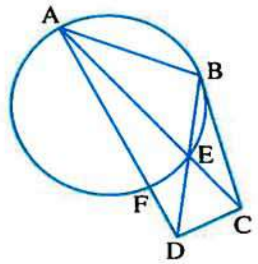
[b] In the opposite figure :

$\overline{BC}$ is a tangent at B ,

E is the midpoint of $\overset{\frown}{BF}$

Prove that :

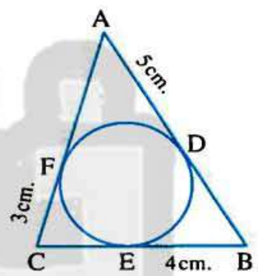
ABCD is a cyclic quadrilateral.



4 **[a] In the opposite figure :**

A circle is drawn touches the sides of a triangle ABC , $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F , AD = 5 cm , BE = 4 cm. , CF = 3 cm.

Find the perimeter of Δ ABC

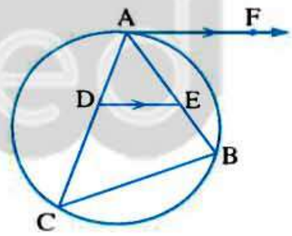


[b] In the opposite figure :

$\overrightarrow{AF}$ is a tangent to the circle at A , $\overrightarrow{AF}$ // $\overrightarrow{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



5 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

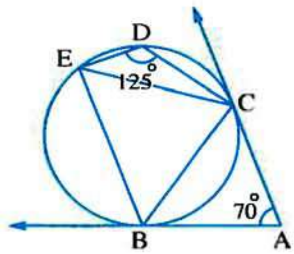
, m (∠ A) = 70° ,

m (∠ CDE) = 125°

Prove that :

1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$



Model examination for the merge students

Answer the following questions in the same paper : (Calculator is allowed)

1 Complete each of the following :

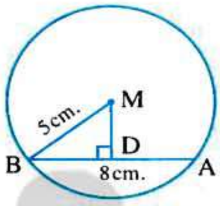
1 The longest chord in the circle is called ……………

2 The straight line passing through the center of the circle and the midpoint of any chord is ……………

3 The two tangent-segments drawn to a circle from a point outside it are …………… in length.

4 **In the opposite figure :**

The length of $\overline{MD}$ = …………… cm.



5 The number of symmetry axes of a circle is ……………

6 If $\overline{AC}$ is a diameter in a circle M , then m $(\overset{\frown}{AC})$ = …………… °

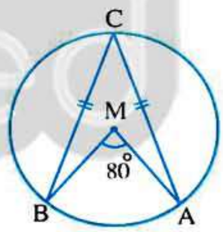
2 Choose the correct answer from those given :

1 If A ∈ the circle M of diameter length 6 cm.
, then MA = …………… cm.

(a) 3 (b) 4
(c) 5 (d) 6

2 **In the opposite figure :**

m (∠ ACB) = ……………



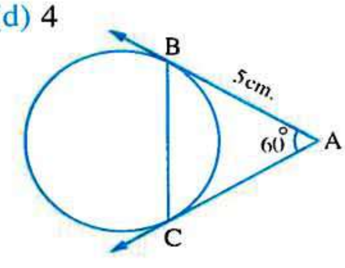
(a) 40° (b) 80°
(c) 90° (d) 180°

3 The number of the common tangents of two distant circles is ……………

(a) 1 (b) 2 (c) 3 (d) 4

4 **In the opposite figure :**

The length of $\overline{BC}$ = …………… cm.



(a) 3 (b) 4
(c) 5 (d) 6

5 The number of circles which can be drawn passing through the endpoints of a line segment $\overline{AB}$ equals ……………

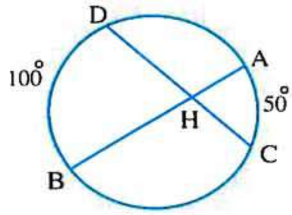
(a) 1 (b) 2 (c) 3 (d) an infinite number.

6 **In the opposite figure :**

m (∠ AHC) = ……………

(a) 25° (b) 50°

(c) 75° (d) 100°



3 Put (✓) for the correct statement , (✗) for the incorrect statement :

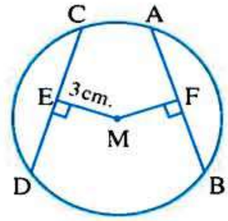
1 If M , N are two touching externally circles with radii lengths are $r_1 = 5$ cm. , $r_2 = 3$ cm. , then MN = 15 cm. ( )

2 **In the opposite figure :**

If AB = CD ,

ME = 3 cm. , then

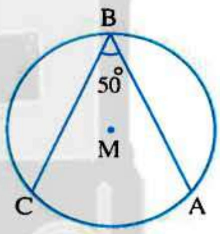
MF = 3 cm. ( )



3 The quadrilateral ABCD is a cyclic quadrilateral if m (∠ A) + m (∠ C) = 90° ( )

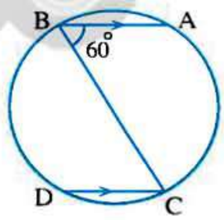
4 **In the opposite figure :**

m $(\overset{\frown}{AC})$ = 100° ( )



5 **In the opposite figure :**

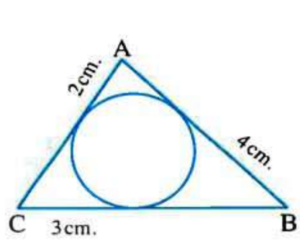
m $(\overset{\frown}{AB})$ + m $(\overset{\frown}{CD})$ = 300° ( )



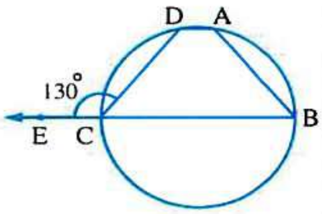
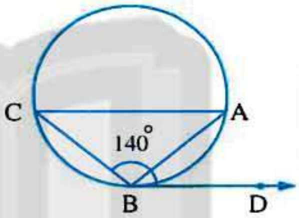
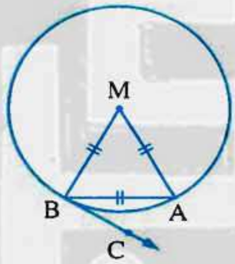
6 **In the opposite figure :**

The perimeter of

Δ ABC = 9 cm. ( )



4 **Join from the column (A) to the suitable one of the column (B) :**

(A)	(B)
1 The measure of the inscribed angle which is drawn in a semicircle equals	• 130°
2 **In the opposite figure :** m (∠ A) = ... 	• 40°
3 **In the opposite figure :** $\overrightarrow{BD}$ is a tangent at B , m (∠ DBC) = 140° , then m (∠ A) = ... 	• 90°
4 The radius of the circumcircle of the vertices of right-angled triangle of hypotenuse length 10 cm. equals .... cm.	• 30°
5 **In the opposite figure :** Δ MAB is an equilateral triangle , $\overrightarrow{BC}$ is a tangent at B , , then m (∠ ABC) = ... 	• 2 : 1
6 The ratio between the measures of the central angle and inscribed angle subtended by the same arc is	• 5

Governorates' Examinations on Geometry

1 Cairo Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1. The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm^2

   (a) 2 (b) 14 (c) 24 (d) 48

2. Two distant circles M and N with radii lengths 6 cm and 8 cm respectively , then MN .... 14 cm.

   (a) < (b) > (c) = (d) ≥

3. The measure of the inscribed angle is .... the measure of the central angle subtended by the same arc.

   (a) half (b) twice (c) quarter (d) third

4. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

5. In the cyclic quad. ABCD , if m (∠ A) = $\frac{1}{2}$ m (∠ C) , then m (∠ A) = ...°

   (a) 20 (b) 30 (c) 60 (d) 120

6. The angle of measure 40° is the complemented angle of the angle of measure .... °

   (a) 320 (b) 140 (c) 60 (d) 50

2 [a] Mention two cases of the cyclic quadrilateral.

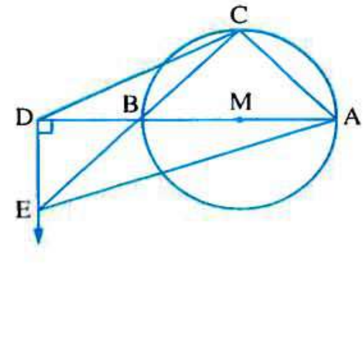
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M , D $\in \overrightarrow{AB}$

, D $\notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$, C $\in \widehat{AB}$

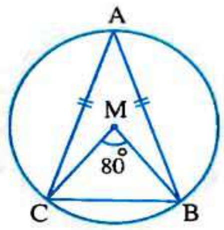
, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

1. **Find :** m (∠ ACB)
2. **Prove that :** The figure ACDE is a cyclic quadrilateral.



3 [a] Find the measure of the arc which represents $\frac{1}{3}$ of the measure of the circle.

[b] **In the opposite figure :**



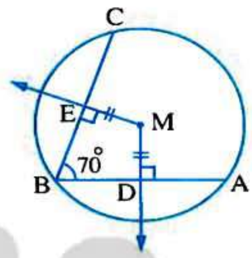
Δ ABC is drawn inside the circle M

, AB = AC , m ($\angle$ BMC) = 80°

Find : 1 m ($\angle$ ABC)

2 The measure of the major arc $\overset{\frown}{BC}$

4 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M , $\overrightarrow{MD} \perp \overline{AB}$

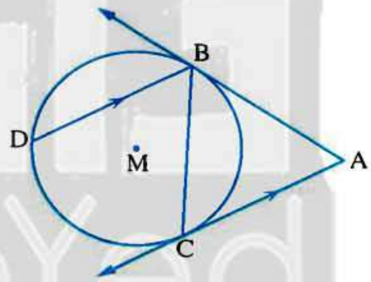
, $\overrightarrow{ME} \perp \overline{CB}$, MD = ME

, m ($\angle$ ABC) = 70°

1 **Find :** m ($\angle$ DME)

2 **Prove that :** AB = CB

[b] **In the opposite figure :**



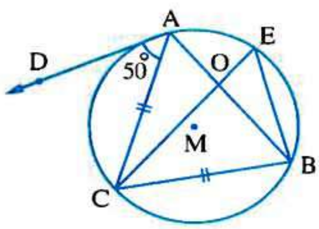
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B and C respectively

, $\overrightarrow{BD}$ // $\overrightarrow{AC}$

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABD

5 [a] Using the geometric tools , draw $\overline{AB}$ with length 6 cm , and then draw a circle passing through the two points A , B with radius length 4 cm. What is the length of the radius of the smallest circle passing through the two points A and B ?

[b] **In the opposite figure :**



A circle M , AC = BC

, $\overrightarrow{AD}$ is a tangent to the circle at A , m ($\angle$ CAD) = 50°

1 **Find :** m ($\angle$ ABC) , m ($\angle$ BEC)

2 **Prove that :**

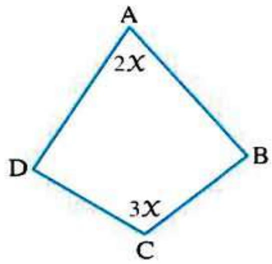
$\overrightarrow{BC}$ is a tangent to the circle passing through the vertices of the triangle BEO

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :



ABCD is a cyclic quadrilateral

, m (∠ A) = 2 x , m (∠ C) = 3 x

, then the value of x = ...°

(a) 20 (b) 30

(c) 32 (d) 36

2 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas equals ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

3 The measure of the inscribed angle in a semicircle equals ....°

(a) 45 (b) 90 (c) 120 (d) 180

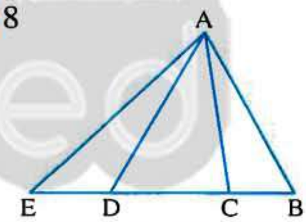
4 The median of the triangle divides its surface into two triangles ....

(a) congruent. (b) equal in area. (c) isosceles. (d) right-angled.

5 If the two circles M , N are touching internally , their radii lengths are 3 cm. , 5 cm. , then MN = ... cm.

(a) 3 (b) 5 (c) 2 (d) 8

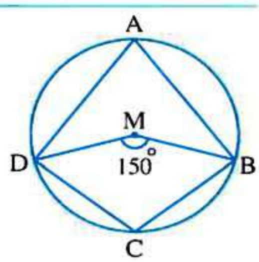
6 The number of triangles in the opposite figure equals ....



(a) 3 (b) 4

(c) 5 (d) 6

2 [a] In the opposite figure :

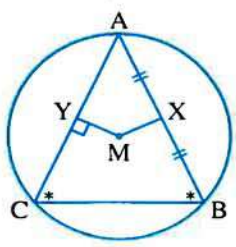


A circle of centre M

, m (∠ BMD) = 150°

Find with proof : m (∠ C)

[b] In the opposite figure :



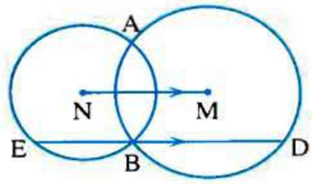
ABC is an inscribed triangle in a circle M

in which m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

3 **[a] In the opposite figure :**

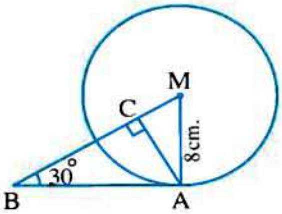


M , N are two intersecting circles at A , B

, $\overleftrightarrow{BD}$ // $\overleftrightarrow{MN}$ and intersects the two circles at D , E

Prove that : DE = 2 MN

[b] In the opposite figure :

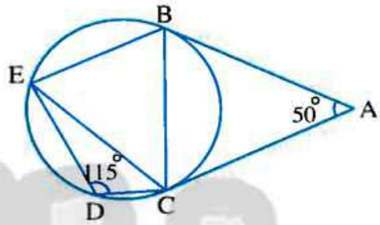


$\overrightarrow{AB}$ is a tangent to the circle M at A

, MA = 8 cm. , m (∠ ABM) = 30° , $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$, $\overline{AC}$

4 **[a] In the opposite figure :**



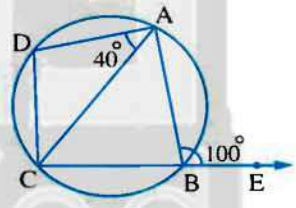
$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B , C

, m (∠ A) = 50° , m (∠ CDE) = 115°

Prove that : 1 $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE

[b] In the opposite figure :

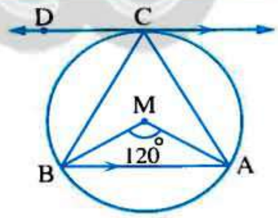


m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$

5 **[a] In the opposite figure :**



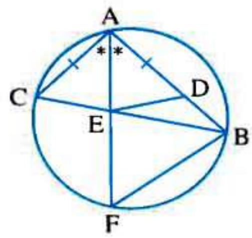
$\overleftrightarrow{CD}$ is a tangent to the circle M at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$

, m (∠ AMB) = 120°

Prove that : Δ CAB is an equilateral triangle.

[b] In the opposite figure :



AC = AD , $\overrightarrow{AE}$ bisects ∠ BAC

and cuts $\overline{BC}$ at E and the circle at F

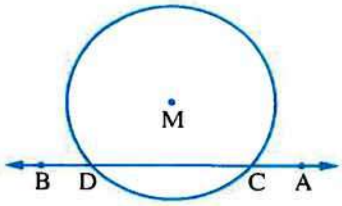
Prove that : BDEF is a cyclic quadrilateral.

3 Alexandria Governorate

Answer the following questions : ***(Calculators are permitted)***

1 Choose the correct answer from those given :

1 In the opposite figure :



$\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\{C , D\}$ (b) $\overline{CD}$

(c) $\overleftrightarrow{CD}$ (d) $\varnothing$

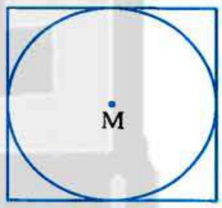
2 $\angle$ A and $\angle$ B are two complementary angles , $\angle$ B and $\angle$ C are two supplementary angles , m ($\angle$ A) = 30° , then m ($\angle$ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

3 If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$ and the radius length of one of them equals 3 cm and MN = 8 cm. , then the radius length of the other circle equals .... cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 In the opposite figure :



If the side length of the square = 10 cm.

, then the surface area of the circle = ... cm^2

(a) 100 π (b) 25 π

(c) 50 π (d) 40 π

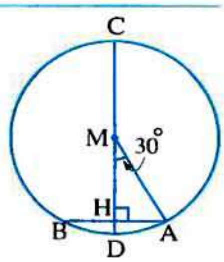
5 A circle can be drawn passing through the vertices of a ....

(a) rhombus (b) parallelogram (c) trapezium (d) rectangle

6 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals .... cm.

(a) 6 (b) 8 (c) 10 (d) 20

2 [a] In the opposite figure :



$\overline{CD}$ is a diameter in the circle M

, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$

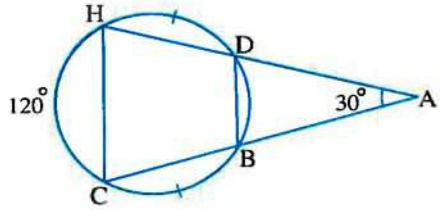
, m ($\angle$ AMD) = 30°

Find : The length of $\overline{CD}$

[b] ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle , $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , if m ($\angle$ AEB) = 70° and m ($\angle$ ADC) = 125°

, **prove that :** AB = AC

3 **[a] In the opposite figure :**



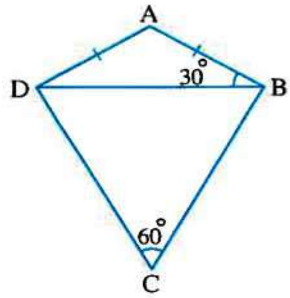
m (∠ A) = 30° , m ($\overset{\frown}{HC}$) = 120°

, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DH}$)

1 **Find :** m ($\overset{\frown}{BD}$) «the minor arc»

2 **Prove that :** AB = AD

[b] In the opposite figure :



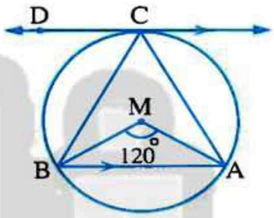
ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

4 **[a] In the opposite figure :**

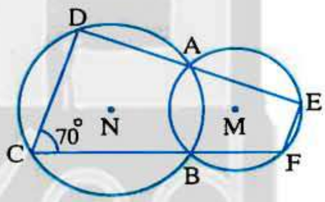


$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.

[b] In the opposite figure :



M and N are two intersecting circles at A and B

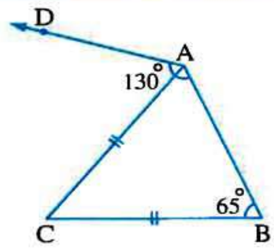
, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D

, $\overleftrightarrow{BC}$ is drawn to intersect the circle M at F and the circle N at C

, m (∠ C) = 70°

Prove that : $\overline{CD}$ // $\overline{EF}$

5 **[a] In the opposite figure :**

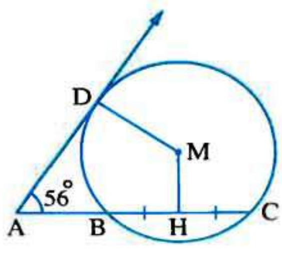


AC = BC , m (∠ ABC) = 65°

, m (∠ DAB) = 130°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

[b] In the opposite figure :



$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, m (∠ A) = 56° and H is the midpoint of $\overline{BC}$

Find with proof : m (∠ DMH)

4 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer :

1 ABC is a triangle in which : $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle$ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 If M and N are two intersecting circles whose radii length are 5 cm and 2 cm. , then MN $\in$ ....

(a) $]3 , 7[$ (b) $[3 , 7[$ (c) $]3 , 7]$ (d) $[3 , 7]$

3 If Δ ABC $\sim \Delta$ XYZ , m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°

(a) 90 (b) 110 (c) 10 (d) 70

4 The measure of the central angle which is opposite to an arc of length $\frac{1}{3}\pi$ r equals ....°

(a) 30 (b) 60 (c) 120 (d) 240

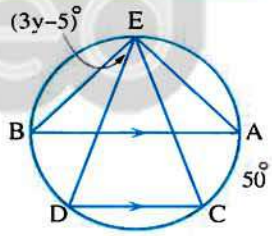
5 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$ where $\overline{BD} \cap \overline{AC} = \{D\}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

6 If ABCD is a cyclic quadrilateral , then m ($\angle$ BAC) = m ($\angle$ ....)

(a) BCA (b) DBA (c) BDC (d) ACD

2 [a] In the opposite figure :



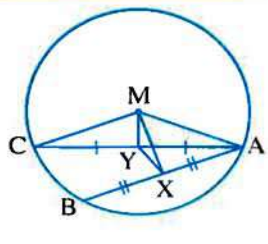
$\overline{AB} \parallel \overline{CD}$, m ($\overset{\frown}{AC}$) = 50°

, m ($\angle$ BED) = (3 y – 5)°

Find : The value of y

[b] Using your geometric tools , draw $\overline{AB}$ with length 4 cm , then draw a circle passing through the two points A and B whose diameter length is 5 cm.
How many circles can be drawn ? (Don't erase the arcs).

3 [a] In the opposite figure :



A circle with centre M

, X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : 1 AXYM is a cyclic quadrilateral.

2 m ($\angle$ MXY) = m ($\angle$ MCY)

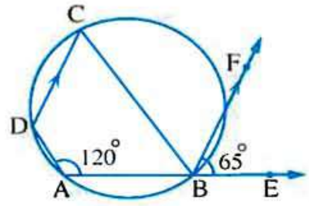
[b] In the opposite figure :

m (∠ A) = 120° , m (∠ EBF) = 65°

, $\overline{DC}$ // $\overrightarrow{BF}$

Find with proof :

1 m (∠ C) 2 m (∠ D)



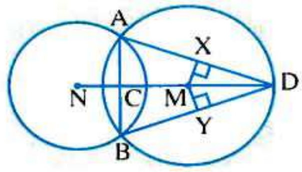
4 **[a] In the opposite figure :**

Circle M ∩ circle N = {A , B}

, $\overleftrightarrow{AB} \cap \overleftrightarrow{MN}$ = {C} , D ∈ $\overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$

Prove that : MX = MY



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$, where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

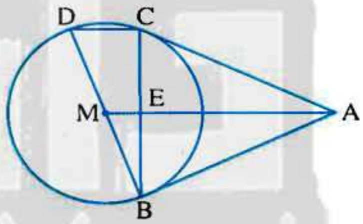
5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AM} \cap \overline{CB}$ = {E}

and $\overline{BD}$ is a diameter of the circle.

Prove that : $\overline{AM}$ // $\overline{CD}$



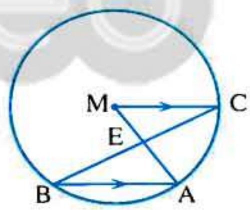
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$

, $\overline{BC} \cap \overline{AM}$ = {E}

Prove that : BE > AE



5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given ones :**

1 A circle can be drawn passing through the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

2 A circle with diameter length 10 cm. , the straight line L is distant from its centre by 5 cm. , then the straight line L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

3 The number of common tangents of two touching circles externally equals ....

(a) zero (b) 1 (c) 2 (d) 3

4 If M , N are two touching circles externally , the lengths of their radii are 2 cm. , 4 cm. respectively , then the area of the circle with diameter $\overline{MN}$ equals .... cm^2

(a) 36π (b) 9π (c) 16π (d) 4π

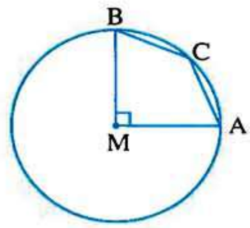
5 **In the opposite figure :**

A circle M , $\overline{MA} \perp \overline{MB}$

, then m ($\angle$ ACB) = ...

(a) 45° (b) 90°

(c) 145° (d) 135°



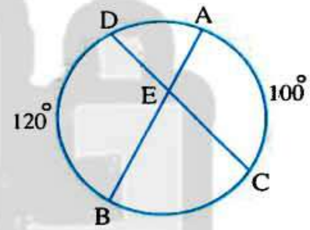
6 **In the opposite figure :**

m ($\overset{\frown}{AC}$) = 100° , m ($\overset{\frown}{DB}$) = 120°

, then m ($\angle$ AEC) = ...

(a) 110° (b) 55°

(c) 70° (d) 100°



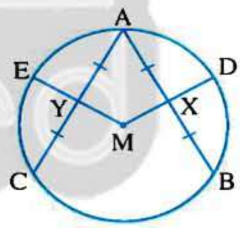
2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two equal chords in circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

Prove that : XD = YE

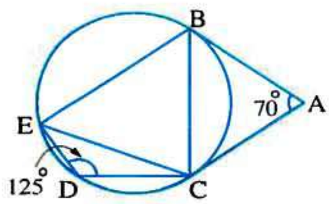


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m ($\angle$ A) = 70° , m ($\angle$ CDE) = 125°

Prove that : $\overrightarrow{BC}$ bisects $\angle$ ABE



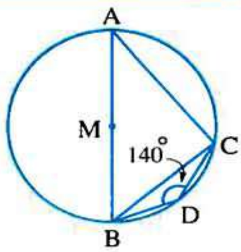
3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m ($\overset{\frown}{BD}$) = m ($\overset{\frown}{DC}$) , m ($\angle$ BDC) = 140°

Find with proof : 1 m ($\angle$ ABC)

2 m ($\overset{\frown}{ABD}$)



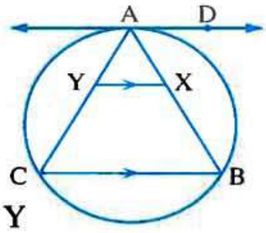
[b] In the opposite figure :

$\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$

, Y $\in \overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y

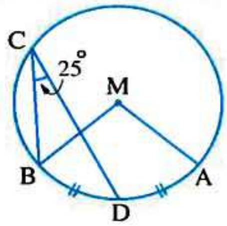


4 **[a] In the opposite figure :**

A circle M , D is the midpoint of $\widehat{AB}$

, m ($\angle$ DCB) = 25°

Find : m ($\angle$ AMB)



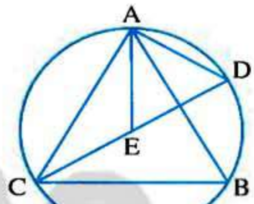
[b] In the opposite figure :

ABC is an equilateral triangle drawn in the circle

, D $\in \widehat{AB}$, E $\in \overline{DC}$, where AD = DE

Prove that : 1 Δ ADE is an equilateral triangle.

2 m ($\angle$ DAB) = m ($\angle$ EAC)



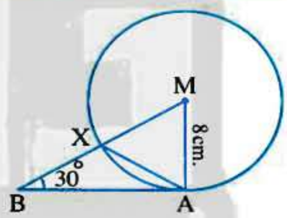
5 **[a] In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M at A

, AM = 8 cm. , m ($\angle$ ABM) = 30°

1 **Find :** The length of $\overline{AB}$

2 **Prove that :** Δ XAB is an isosceles triangle.

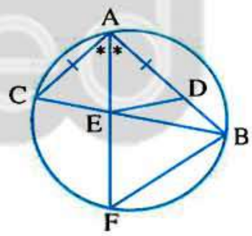


[b] In the opposite figure :

AD = AC

, $\overrightarrow{AF}$ bisects $\angle$ BAC

Prove that : DBFE is a cyclic quadrilateral.



6 El-Monofia Governorate

Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

1 The axis of symmetry of a circle is ....

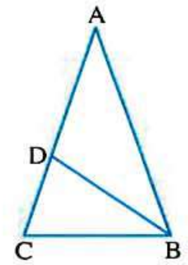
(a) the diameter. (b) the chord.

(c) the straight line passing through the center. (d) the tangent.

2 XYZ is a triangle. If $(XY)^2 - (YZ)^2 > (XZ)^2$, then $\angle$ Y is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

3 **In the opposite figure :**



If AB = AC , BC = BD = AD

, then m ($\angle$ A) = ...°

(a) 30 (b) 36

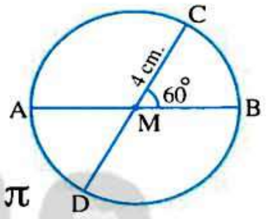
(c) 45 (d) 72

4 ABCD is a cyclic quadrilateral in which m ($\angle$ A) = 2 m ($\angle$ C)

, then m ($\angle$ C) = ...°

(a) 30 (b) 60 (c) 90 (d) 120

5 **In the opposite figure :**



A circle M , MC = 4 cm.

, m ($\angle$ CMB) = 60°

, then the length of $\widehat{BD}$ = ... cm.

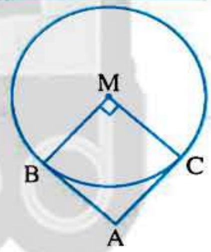
(a) 4π (b) 8π (c) $\frac{8}{3}\pi$ (d) 16π

6 If Y $\in \overline{XZ}$ and XY = 2 YZ , then the area of the square drawn on $\overline{XY}$ = ...

The area of the square drawn on $\overline{XZ}$

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 2 (d) $\frac{1}{2}$

2 [a] **In the opposite figure :**

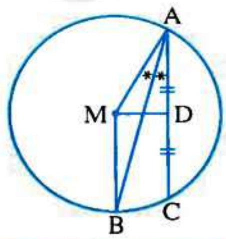


$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m ($\angle$ BMC) = 90°

Prove that : ABMC is a square.

[b] **In the opposite figure :**



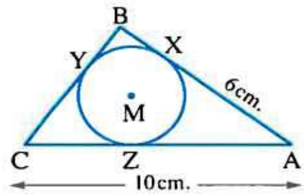
$\overline{AC}$ is a chord in the circle M

, $\overrightarrow{AB}$ bisects $\angle$ CAM

, D is the midpoint of $\overline{AC}$

Prove that : $\overline{DM} \perp \overline{MB}$

3 [a] **In the opposite figure :**



$\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are tangents to the circle M at X

, Y and Z respectively , AC = 10 cm.

, AX = 6 cm. and the perimeter of Δ ABC = 24 cm.

1 **Find :** The length of $\overline{AB}$

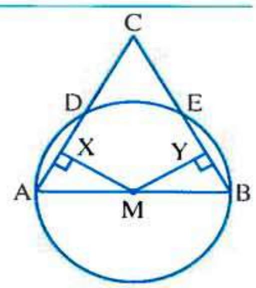
2 Determine the type of Δ ABC according to the measures of its angles.

[b] ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$ where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 The figure BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$

4 **[a] In the opposite figure :**



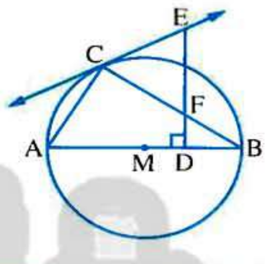
$\overline{AB}$ is a diameter in the circle M

, CA = CB , $\overline{MX} \perp \overline{DA}$

, $\overline{MY} \perp \overline{EB}$

Prove that : CD = CE

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

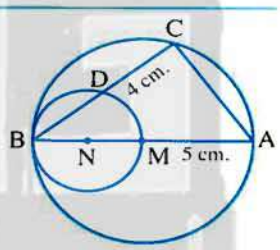
, $\overleftrightarrow{EC}$ is a tangent to the circle M at C , $\overline{ED} \perp \overline{AB}$

, where $\overline{ED} \cap \overline{CB} = \{F\}$

Prove that : 1 The figure ADFC is a cyclic quadrilateral.

2 Δ ECF is an isosceles triangle.

5 **[a] In the opposite figure :**

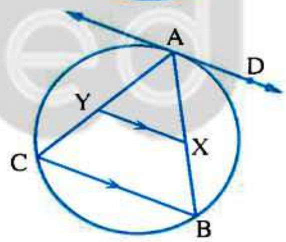


M , N are two circles touching internally at B

, AM = 5 cm. , CD = 4 cm.

Find with proof : The length of $\overline{AC}$

[b] In the opposite figure :



ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A , $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 A square whose diagonal length is 10 cm. , then its surface area equals …………… cm^2.

(a) 40 (b) 50 (c) 80 (d) 100

2 ABC is a triangle in which $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ BAC is ……………

(a) acute. (b) obtuse. (c) right. (d) straight.

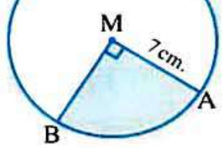
3 M and N are two intersecting circles at two points and the two radii lengths are 3 cm. and 5 cm. , then MN $\in$ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

4 ABCD is a cyclic quadrilateral in which m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...°

(a) 90 (b) 45 (c) 135 (d) 120

5 **In the opposite figure :**

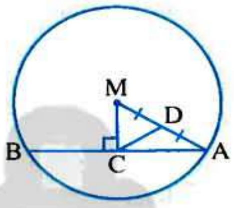


$\overline{MA}$, $\overline{MB}$ are two radii perpendicular in the circle M whose radius length is 7 cm.

, then the perimeter of the shaded part = ... cm. $\left(\pi = \frac{22}{7}\right)$

(a) 14 (b) 11 (c) $38\frac{1}{2}$ (d) 25

6 **In the opposite figure :**



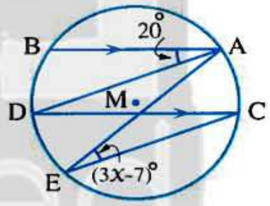
$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, D is the midpoint of $\overline{MA}$, CD = 3 cm.

, then the surface area of the circle M = ... π cm².

(a) 3 (b) 6 (c) 9 (d) 36

2 [a] **In the opposite figure :**

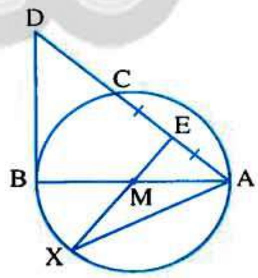


$\overline{AB}$ // $\overline{CD}$, m (∠ BAD) = 20°

, m (∠ AEC) = $(3X - 7)°$

What is the value of X ?

[b] **In the opposite figure :**

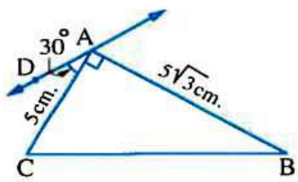


$\overline{AB}$ is a diameter in the circle M , $\overline{BD}$ is a tangent-segment to the circle M at B , E is the midpoint of $\overline{AC}$ and $\overrightarrow{EM}$ intersects the circle M at X

Prove that : **1** The figure MEDB is a cyclic quadrilateral.

2 m (∠ BAX) = $\frac{1}{2}$ m (∠ D)

3 [a] **In the opposite figure :**



ABC is a right-angled triangle at A

, AC = 5 cm. , AB = $5\sqrt{3}$ cm.

, m (∠ DAC) = 30°

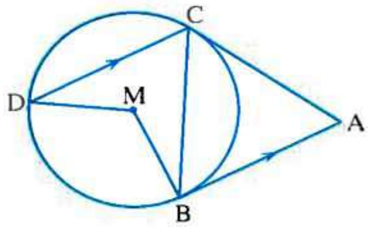
Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$

Prove that : $\overrightarrow{CB}$ bisects $\angle$ ACD



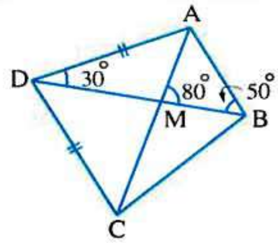
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AC} \cap \overline{BD} = \{M\}$, DA = DC

, m ($\angle$ ADM) = 30° , m ($\angle$ AMB) = 80°

, m ($\angle$ ABD) = 50°

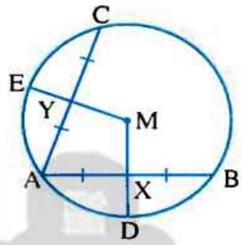
Prove that : The figure ABCD is a cyclic quadrilateral.



[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overrightarrow{MX}$ intersects the circle M at D , $\overrightarrow{MY}$ intersects the circle M at E

Prove that : XD = YE



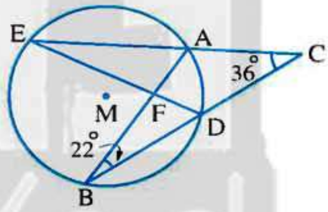
5 **[a] In the opposite figure :**

$\overrightarrow{EA} \cap \overrightarrow{BD} = \{C\}$

, m ($\angle$ C) = 36°

, m ($\angle$ ABD) = 22°

Find with the proof : m $(\widehat{BE})$



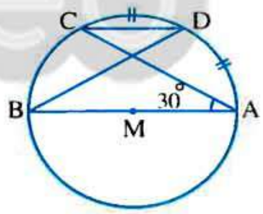
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m ($\angle$ CAB) = 30° , m $(\widehat{AD})$ = m $(\widehat{DC})$

1 **Find with the proof :** m ($\angle$ CDB)

2 **Prove that :** $\overline{DC}$ // $\overline{AB}$



8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **[a] Choose the correct answer from the given ones :**

1 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = ... cm.

(a) 12 π (b) 6 π (c) 24 π (d) 10 π

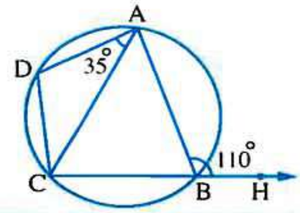
2 M and N are two circles whose radii lengths are 6 cm. , 8 cm. and MN = 14 cm. , then the two circles are ....

(a) intersecting. (b) distant.
(c) one inside the other. (d) touching externally.

3 The inscribed angle drawn in a semicircle is .... angle.

(a) an acute (b) a straight (c) a right (d) an obtuse

[b] In the opposite figure :



m (∠ ABH) = 110°
, m (∠ CAD) = 35°
Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$

2 **[a] Choose the correct answer from the given ones :**

1 A chord is of length 8 cm. in a circle of diameter length 10 cm. , then the chord is at .... from the center of the circle.

(a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.

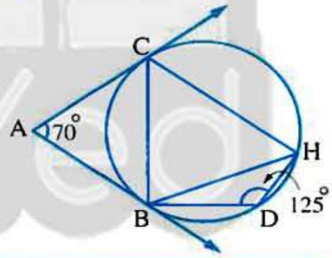
2 The number of common tangents of two circles touching internally is ....

(a) 1 (b) 3 (c) 2 (d) 0

3 ABCD is a cyclic quadrilateral , m (∠ A) = 2 m (∠ C) , then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

[b] In the opposite figure :

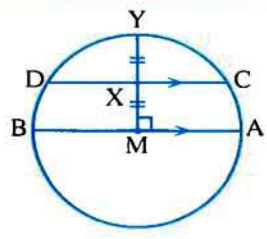


$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B , C
, m (∠ A) = 70° , m (∠ D) = 125°

1 **Find :** m (∠ ABC)

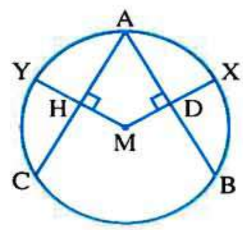
2 **Prove that :** CB = BH

3 **[a] In the opposite figure :**



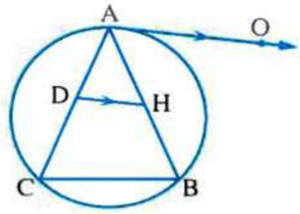
$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ // $\overline{AB}$
, X is the midpoint of $\overline{MY}$
, $\overline{MY} \perp \overline{AB}$
Find : m $(\overset{\frown}{AC})$, m $(\overset{\frown}{CY})$

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two equal chords in the circle M
, $\overrightarrow{MD} \perp \overline{AB}$ and cuts the circle at X
, $\overrightarrow{MH} \perp \overline{AC}$ and cuts the circle at Y
Prove that : XD = HY

4 **[a] In the opposite figure :**

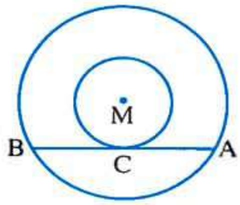


$\overrightarrow{AO}$ is a tangent to the circle at A

, $\overrightarrow{AO}$ // $\overline{DH}$

Prove that : DHBC is a cyclic quadrilateral.

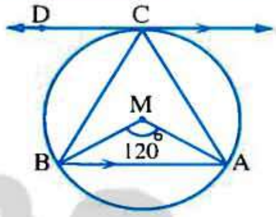
[b] In the opposite figure :



$\overline{AB}$ is a chord in the greater circle M and touches the smaller circle at C , if AB = 14 cm.

, find the area of the part included between the two circles.

5 **[a] In the opposite figure :**



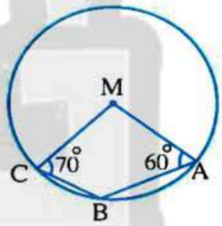
The circle M passes through the vertices of the triangle ABC

, m (∠ AMB) = 120°

, $\overleftrightarrow{CD}$ is a tangent to the circle M at C , $\overleftrightarrow{CD}$ // $\overline{AB}$

Prove that : Δ ABC is equilateral.

[b] In the opposite figure :



m (∠ MAB) = 60°

, m (∠ MCB) = 70°

Find : m (∠ AMC)

9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

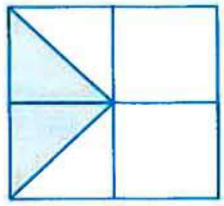
1 **Choose the correct answer from those given :**

1. The least number of acute angles at any triangle equals ....
   (a) zero (b) 1 (c) 2 (d) 3
2. The measure of the central angle drawn in $\frac{1}{3}$ circle equals ....°
   (a) 240 (b) 120 (c) 60 (d) 30
3. ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2 + 5$, then ∠ B is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
4. Which of the following figures is a cyclic quadrilateral ?
   (a) The square. (b) The rhombus.
   (c) The parallelogram. (d) The trapezium.

5 If AB = 8 cm. , then the length of the radius of the smallest circle can be drawn passing through the two points A and B equals ………….. cm.

(a) 1 (b) 2 (c) 3 (d) 4

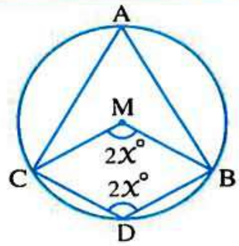
6 **In the opposite figure :**



A square consists of congruent squares , then the area of the shaded part = ………….. the figure area.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

2 **[a] In the opposite figure :**



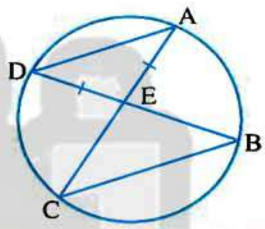
$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M

, $D \in \widehat{BC}$

, m (∠ BMC) = m (∠ BDC) = $(2x)^\circ$

Find with proof : m (∠ A)

[b] In the opposite figure :

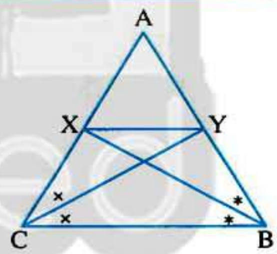


$\overline{AC} \cap \overline{BD} = \{E\}$

, EA = ED

Prove that : EB = EC

3 **[a] In the opposite figure :**



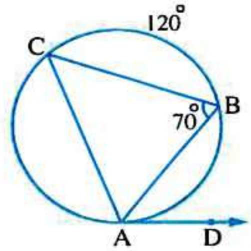
ABC is a triangle in which AB = AC

, $\overrightarrow{BX}$ bisects ∠ ABC and intersects $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects ∠ ACB and intersects $\overline{AB}$ at Y

Prove that : BCXY is a cyclic quadrilateral.

[b] In the opposite figure :



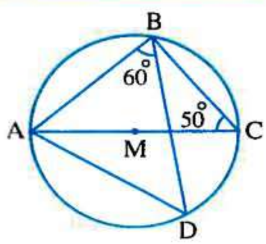
$\overrightarrow{AD}$ is a tangent to the circle at A

, m (∠ B) = 70°

, m $(\widehat{BC})$ = 120°

Find : m (∠ DAB)

4 **[a] In the opposite figure :**



$\overline{AC}$ is a diameter of the circle M

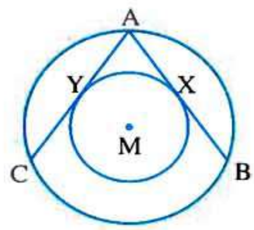
, m (∠ C) = 50° , m (∠ ABD) = 60°

Find : m (∠ CBD) , m (∠ BAD)

[b] **In the opposite figure :**

Two concentric circles at M , $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and two tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC



5 [a] **In the opposite figure :**

Two circles are touching externally at C

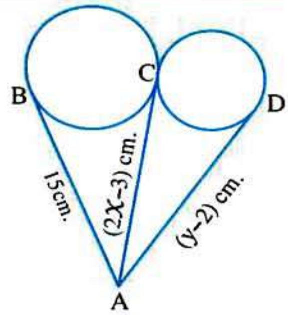
, $\overline{AD}$ is a tangent-segment to the smaller circle at D

, $\overline{AB}$ is a tangent-segment to the greater circle at B

If AD = (y – 2) cm.

, AC = (2 X – 3) cm. , AB = 15 cm.

Find with proof : The value of each of X and y

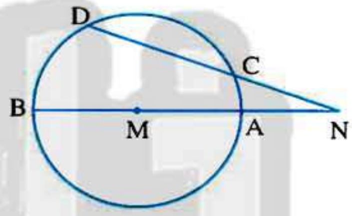


[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NB > ND



10 Suez Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The inscribed angle drawn in a semicircle is ……………

(a) reflex. (b) right. (c) obtuse. (d) acute.

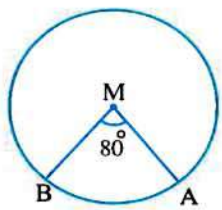
2 **In the opposite figure :**

If M is a circle , m (∠ AMB) = 80°

then m ($\widehat{AB}$) = ……………°

(a) 40 (b) 80

(c) 160 (d) 90



3 If the two circles M , N are touching externally , the length of the radius of one of them is 3 cm. , MN = 8 cm. , then the length of the radius of the other circle is …………… cm.

(a) 5 (b) 6 (c) 11 (d) 16

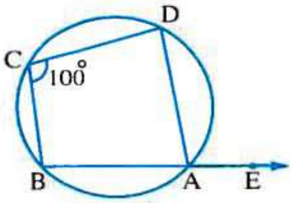
4 **In the opposite figure :**

$E \in \overrightarrow{BA}$, m ($\angle$ C) = 100°

, then m ($\angle$ DAE) = ... °

(a) 80 (b) 60

(c) 100 (d) 200



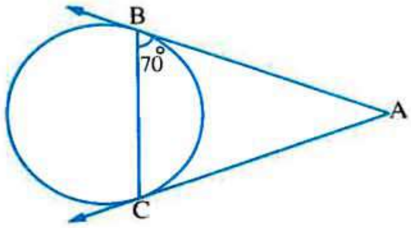
5 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C , m ($\angle$ ABC) = 70°

, then m ($\angle$ A) = ... °

(a) 80 (b) 70

(c) 60 (d) 40



6 The area of the circle = ...

(a) $2 \pi r$ (b) πr^2 (c) $2 \pi r^2$ (d) πr

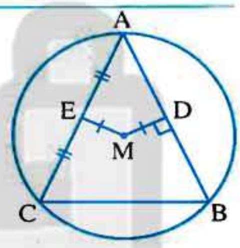
2 **[a] In the opposite figure :**

If M is a circle , $\overline{MD} \perp \overline{AB}$

, E is the midpoint of $\overline{AC}$

, MD = ME

, prove that : AB = AC

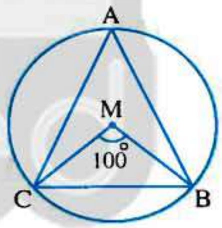


[b] In the opposite figure :

If M is a circle , m ($\angle$ BMC) = 100°

, find : **1** m ($\angle$ A)

2 m ($\angle$ MBC)



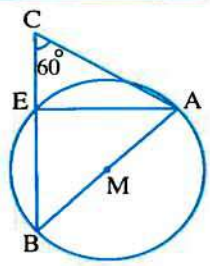
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, $C \in \overrightarrow{BE}$, m ($\angle$ ACE) = 60°

Find : **1** m ($\angle$ AEB)

2 m ($\angle$ CAE)



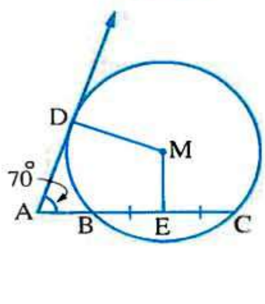
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B , C

, E is the midpoint of $\overline{BC}$, m ($\angle$ A) = 70°

Find : m ($\angle$ DME)



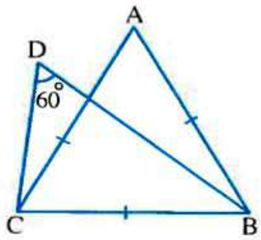
4 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

ABC is an equilateral triangle

, m (∠ D) = 60°

Prove that : ABCD is a cyclic quadrilateral.

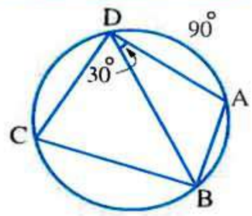


5 **[a] In the opposite figure :**

In the circle , m (∠ ADB) = 30°

, m $(\overset{\frown}{AD})$ = 90°

Find : **1** m $(\overset{\frown}{AB})$ **2** m (∠ DCB)



[b] In the opposite figure :

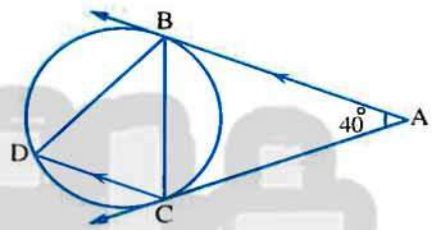
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents

to the circle at B and C

, $\overrightarrow{AB}$ // $\overline{CD}$, m (∠ A) = 40°

1 **Find :** m (∠ ABC)

2 **Prove that :** BC = BD



11 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 M and N are two intersecting circles. The two radii lengths are 3 cm. and 5 cm. respectively , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 5 (d) 10

3 The longest chord in the circle is called a ....

(a) chord. (b) diameter. (c) tangent. (d) radius.

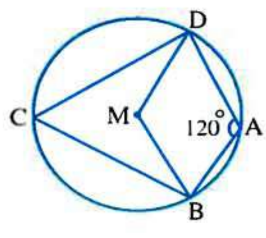
4 **In the opposite figure :**

If m (∠ A) = 120°

, then m (∠ DMB) = ...

(a) 180° (b) 120°

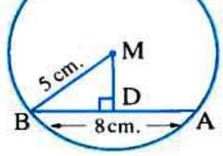
(c) 90° (d) 60°



5 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc is ……………

(a) 4 : 2 (b) 2 : 4 (c) 3 : 2 (d) 2 : 3

6 **In the opposite figure :**



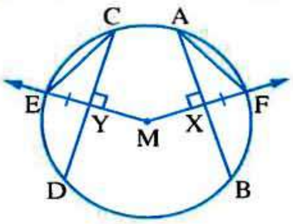
AB = 8 cm. , MB = 5 cm.

, then MD = ……………

(a) 5 cm. (b) 3 cm.

(c) 4 cm. (d) 2 cm.

2 [a] **In the opposite figure :**



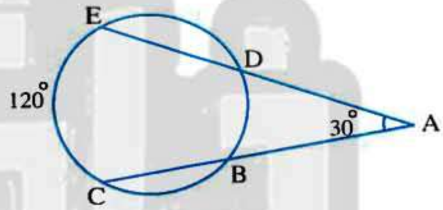
$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY

Prove that : 1 AB = CD 2 AF = CE

[b] **In the opposite figure :**



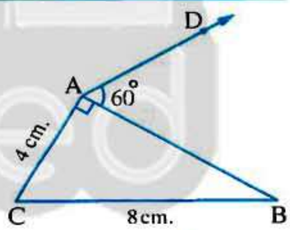
$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

, m $(\widehat{CE})$ = 120°

, m (∠ A) = 30°

Find : m $(\widehat{BD})$

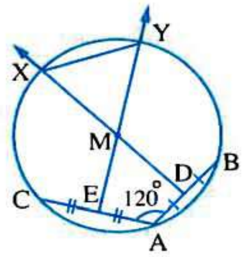
3 [a] **Using the given data , prove that :**



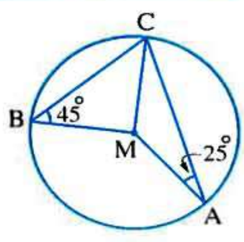
$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

[b] **Using the given data , prove that :**

The triangle XYM is an equilateral triangle.



4 [a] **In the opposite figure :**



A circle with center M

, m (∠ MAC) = 25°

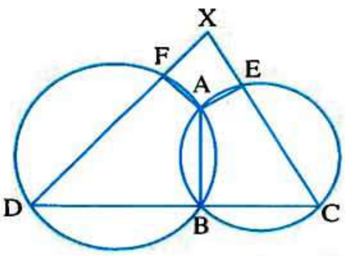
, m (∠ MBC) = 45°

Find : m (∠ AMB)

[b] In the opposite figure :

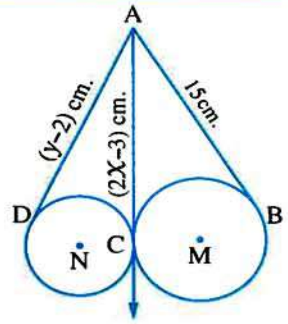
Two intersecting circles at A and B , $\overline{CD}$ passes through the point B and intersects the two circles at C and D , $\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$

Prove that : The figure AFXE is a cyclic quadrilateral.



5 **[a] Using the given data , find :**

The values of the symbols x and y

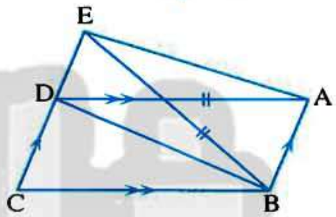


[b] In the opposite figure :

ABCD is a parallelogram

, $E \in \overrightarrow{CD}$ where BE = AD

Prove that : The figure ABDE is a cyclic quadrilateral.



12 Damietta Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1 The length of the projection of a line segment on a given straight line .... the length of the line segment.

(a) > (b) ≤ (c) ≥ (d) <

2 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number

3 If a square is of side length 6 cm. , then the square of its diagonal length is .... cm^2

(a) 36 (b) 12 (c) 72 (d) $6\sqrt{2}$

4 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 5 (c) 6 (d) 10

5 If M , N are two touching circles internally , their radii lengths are 7 cm. , 10 cm. , then MN = ... cm.

(a) 3 (b) 17 (c) 7 (d) 10

6 If $\Delta XYZ \sim \Delta ABC$, m ($\angle$ Y) = 60° and m ($\angle$ C) = 40° , then m ($\angle$ X) = ...°

(a) 40 (b) 80 (c) 100 (d) 120

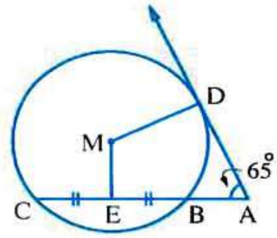
2 **[a] In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M , $\overline{AC}$ intersects the circle at B , C

, E is the midpoint of $\overline{BC}$

, m ($\angle$ A) = 65°

Find : m ($\angle$ DME)



[b] If the length of $\overline{AB}$ = 6 cm. , draw a circle of radius length 4 cm. that passes through A , B

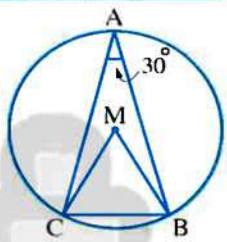
How many circles can be drawn ? (Don't remove the arcs).

3 **[a] In the opposite figure :**

A circle M , m ($\angle$ A) = 30°

1 **Find :** m ($\angle$ BMC)

2 **Prove that :** MBC is an equilateral triangle.



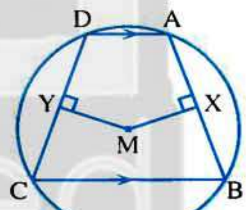
[b] In the opposite figure :

A circle M , $\overline{AD}$ // $\overline{BC}$

, $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{DC}$

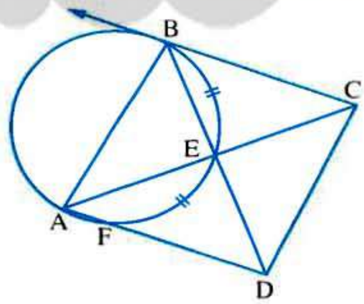
Prove that : MX = MY



4 **[a] In the opposite figure :**

$\overrightarrow{CB}$ is a tangent , m ($\widehat{BE}$) = m ($\widehat{EF}$)

Prove that : ABCD is a cyclic quadrilateral.



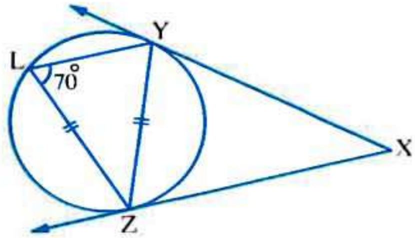
[b] In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, YZ = LZ , m ($\angle$ L) = 70°

1 **Find with proof :** m ($\angle$ X)

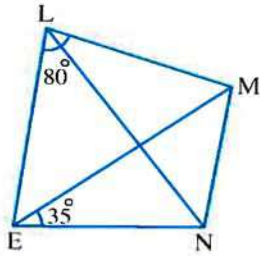
2 **Prove that :** $\overline{XZ}$ // $\overline{YL}$



5 **[a]** ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

[b] In the opposite figure :



LMNE is a cyclic quadrilateral , m (∠ MEN) = 35°

, m (∠ MLE) = 80°

Find with proof :

1 m (∠ MLN)

2 m (∠ EMN)

13 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The triangle contains two .... angles at least.
(a) acute (b) obtuse (c) right (d) reflex

2 ABCD is a rhombus in which m (∠ ACB) = 32° , then m (∠ D) = ...
(a) 32° (b) 64° (c) 116° (d) 26°

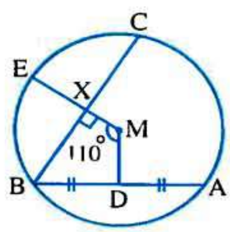
3 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its center.
(a) 6 (b) 12 (c) 3 (d) 2

4 If M , N are two touching circles internally their radii lengths are 8 cm. , 3 cm. , then MN = ... cm.
(a) 3 (b) 5 (c) 7 (d) 11

5 The triangle whose side lengths are 5 cm. , 7 cm. and 8 cm. is .... triangle.
(a) obtuse-angled. (b) acute-angled. (c) right-angled. (d) equilateral.

6 The number of common tangents to two touching circles externally is ....
(a) 0 (b) 1 (c) 2 (d) 3

2 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M

, which has radius length of 10 cm.

, $\overrightarrow{MX} \perp \overline{BC}$ intersecting $\overline{BC}$ at X and intersecting the circle M at E

, D is the midpoint of $\overline{AB}$, BC = 16 cm.

, m (∠ DMX) = 110°

Find : 1 The length of $\overline{XE}$ 2 m (∠ ABC)

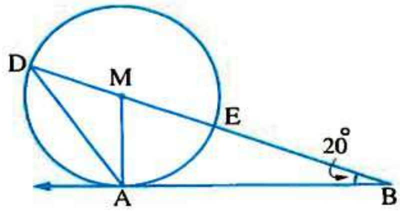
[b] In the opposite figure :

B is a point outside the circle M

, $\overrightarrow{BA}$ is a tangent to the circle M at A

, $\overrightarrow{BM}$ intersects the circle at E and D , m ($\angle$ B) = 20°

Find with proof : m ($\angle$ ADB)

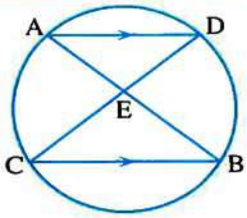


3 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, $\overline{AD}$ // $\overline{CB}$

Prove that : EA = ED



[b] In the opposite figure :

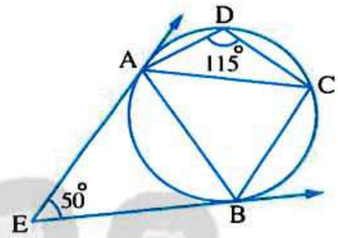
$\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A , B

, m ($\angle$ AEB) = 50°

, m ($\angle$ ADC) = 115°

Prove that :

$\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E

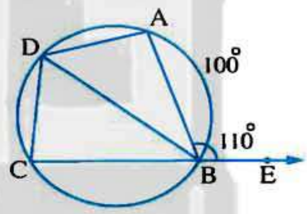


4 **[a] In the opposite figure :**

E $\in \overrightarrow{CB}$, m ($\widehat{AB}$) = 100°

, m ($\angle$ ABE) = 110°

Find with proof : m ($\angle$ BDC)



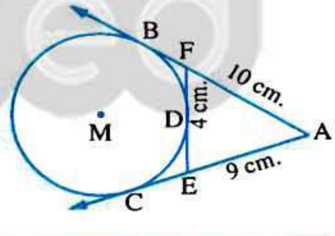
[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B , C

, $\overline{FE}$ is a tangent-segment at D , DF = 4 cm.

, AF = 10 cm. , AE = 9 cm.

Find with proof : The length of $\overline{EC}$



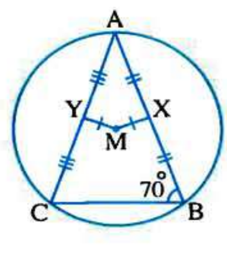
5 **[a] In the opposite figure :**

ABC is an inscribed triangle inside the circle M

, MX = MY , X and Y are the midpoints of $\overline{AB}$

, $\overline{AC}$ respectively , m ($\angle$ B) = 70°

Find with proof : m ($\angle$ A)



[b] ABC is an inscribed triangle in a circle where AB > AC and D $\in \overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects $\angle$ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then MN ∈ ....

(a)]8 , ∞[(b)]2 , ∞[(c)]0 , 2[(d)]2 , 8[

2 ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) equals ....

(a) 25° (b) 20° (c) 110° (d) 100°

3 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 130° (b) 90° (c) 50° (d) 180°

4 The slope of the straight line $3x + 2y = 1$ is ....

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$

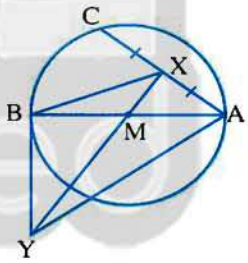
5 The measurement of any angle of the regular hexagon is ....

(a) 90° (b) 108° (c) 120° (d) 135°

6 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then ∠ B is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

2 [a] In the opposite figure :

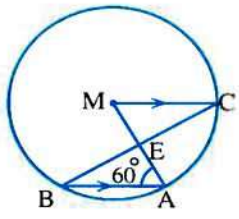


$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersects the tangent to the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.

[b] In the opposite figure :



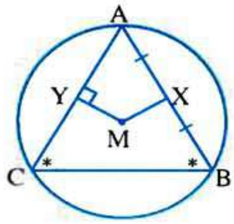
$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

, m (∠ A) = 60°

Find : m (∠ B)

3 [a] In the opposite figure :



The triangle ABC is inscribed in the circle M

, in which : m (∠ B) = m (∠ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

[b] In the opposite figure :

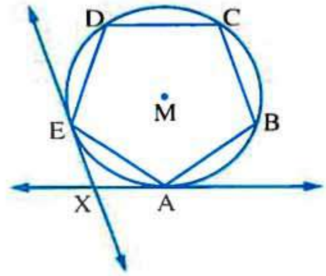
ABCDE is a regular pentagon inscribed in a circle M

, $\overleftrightarrow{AX}$ is a tangent to the circle at A

, $\overleftrightarrow{EX}$ is a tangent to the circle at E

where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$

Find : 1 m $(\widehat{AE})$ 2 m $(\angle AXE)$



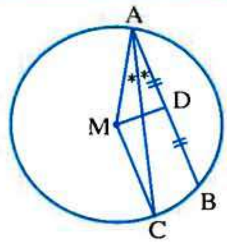
4 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overrightarrow{AC}$ bisects $\angle$ BAM and intersects the circle M at C

If D is the midpoint of $\overline{AB}$

, prove that : $\overline{DM} \perp \overline{CM}$



[b] $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle M , $\overrightarrow{CM}$ intersects the circle M at X and Y and intersects $\overleftrightarrow{BD}$ at E **Prove that :** CX = YE

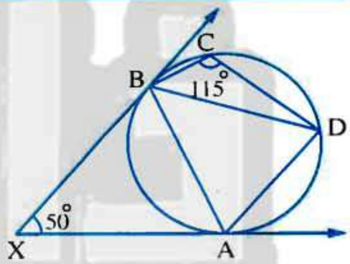
5 **[a] In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m $(\angle AXB) = 50°$, m $(\angle DCB) = 115°$

Prove that : 1 $\overrightarrow{AB}$ bisects $\angle$ DAX

2 BD = BA

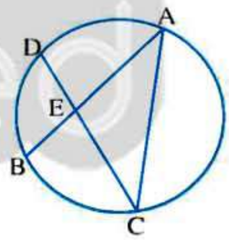


[b] In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



15 El-Fayoum Governorate

Answer the following questions : ***(Using calculators is allowed)***

1 **Choose the correct answer :**

1 If M is a circle of diameter length 8 cm. , the straight line L is far from the centre M of the circle by 4 cm. , then the straight line L is ....

(a) a secant to the circle in two points. (b) outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry of the circle.

2 If m_1 , m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 = m_2$ (b) $m_1 \times m_2 = -1$ (c) $m_1 \times m_2 = 1$ (d) $m_1 + m_2 = -1$

3 The centre of the circle that passes through the vertices of the triangle is the intersection point of ....

(a) the bisectors of its interior angles.
(b) the bisectors of its exterior angles.
(c) its altitudes.
(d) the axes of its sides.

4 ABC is a right-angled triangle at B , m (∠ C) = 30° , AC = 12 cm. , then AB = ... cm.

(a) 24 (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) 6

5 Which of the following figures is a cyclic quadrilateral ?

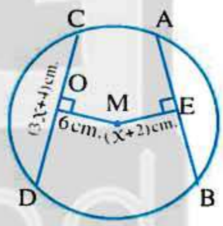
(a) The rectangle. (b) The trapezium. (c) The rhombus. (d) The parallelogram.

6 A trapezium in which the lengths of the two parallel bases are 4 cm. and 12 cm. and its height is 9 cm. , then its area = ... cm^2.

(a) 25 (b) 36 (c) 72 (d) 144

2 **[a] In the opposite figure :**

AB = CD , MO = 6 cm.
, ME = (X + 2) cm.
, CD = (3 X + 4) cm.
Find : The value of X , CD



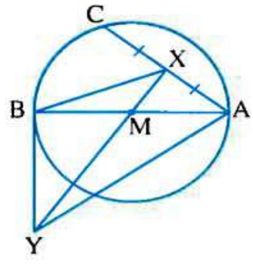
[b] ABC is a triangle drawn inside a circle M , m (∠ AMB) = 90° , m (∠ BMC) = 130°
Find : The measures of the angles of Δ ABC

3 **[a]** A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AM}$ intersects the circle M at C and D respectively , m (∠ A) = 40°

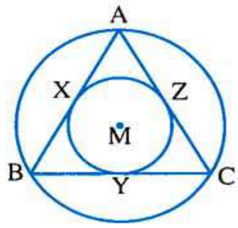
Find with proof : m (∠ BDC)

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M
, X is the midpoint of $\overline{AC}$
and $\overrightarrow{XM}$ intersects the tangent to the circle at B at Y
Prove that : The figure AXBY is a cyclic quadrilateral.



4 **[a] In the opposite figure :**



Two concentric circles with centre M

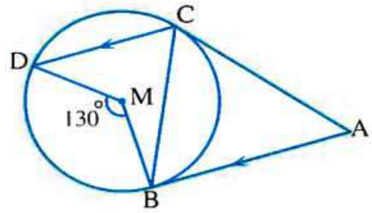
, the radii lengths of them are 4 cm. and 2 cm.

, Δ ABC is an inscribed triangle inside the greater circle

, and its sides touch the smaller circle at X , Y , Z

Prove that : Δ ABC is an equilateral triangle , and calculate its area.

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB} \parallel \overline{CD}$

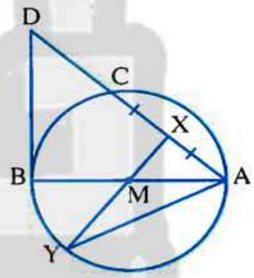
, m ($\angle$ BMD) = 130°

Prove that : $\overrightarrow{CB}$ bisects $\angle$ ACD

5 **[a]** Δ ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$ and Y $\in \overline{AC}$, where $\overline{XY} \parallel \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M ,

X is the midpoint of $\overline{AC}$, $\overrightarrow{BD}$ is a tangent to the circle at B , $\overrightarrow{XM}$ intersects the circle at Y

Prove that : **1** XMBD is a cyclic quadrilateral.

2 m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ D)

16 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The symmetry axis of the common chord $\overline{AB}$ of the two intersecting circles M , N is ....

(a) $\overleftrightarrow{MA}$ (b) $\overleftrightarrow{MB}$ (c) $\overleftrightarrow{MN}$ (d) $\overleftrightarrow{NA}$

2 ABC is a triangle in which : $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ B is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 In the cyclic quadrilateral , each two opposite angles are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) alternate.

4 The area of a triangle is 35 cm^2 and its height is 7 cm. , then the length of its base equals .... cm.

(a) 5 (b) 7 (c) 10 (d) 20

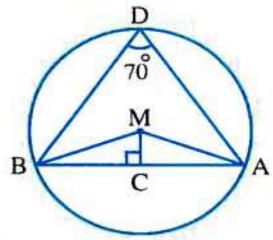
5 The measure of the inscribed angle which is drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 180°

6 The area of a square is 100 cm^2 , then its perimeter = ... cm.

(a) 10 (b) 30 (c) 40 (d) 50

2 **[a] In the opposite figure :**

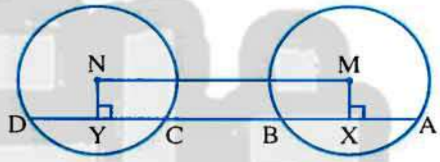


$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, m (∠ ADB) = 70°

Find : m (∠ AMC)

[b] In the opposite figure :

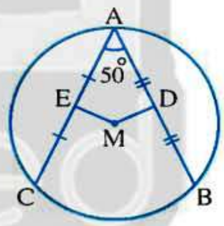


M and N are two congruent circles

, AB = CD , $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.

3 **[a] In the opposite figure :**



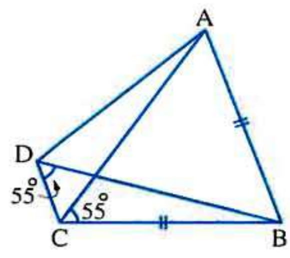
$\overline{AB}$ and $\overline{AC}$ are two chords

in the circle M , D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ and m (∠ BAC) = 50°

Find : m (∠ DME)

[b] In the opposite figure :



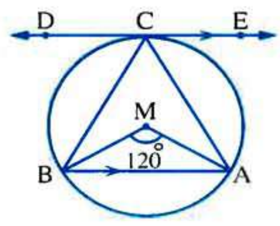
AB = BC

, m (∠ ACB) = 55°

and m (∠ BDC) = 55°

Prove that : The figure ABCD is a cyclic quadrilateral.

4 **[a] In the opposite figure :**



$\overleftrightarrow{ED}$ is a tangent to the circle M at C

, $\overleftrightarrow{ED}$ // $\overline{AB}$ and m (∠ AMB) = 120°

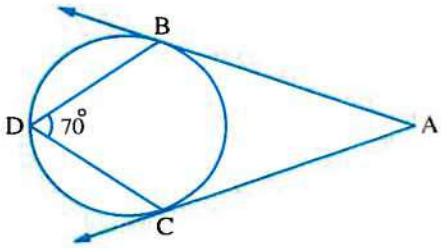
Prove that : The triangle CAB is an equilateral triangle.

[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ BDC) = 70°

Find : m (∠ A)

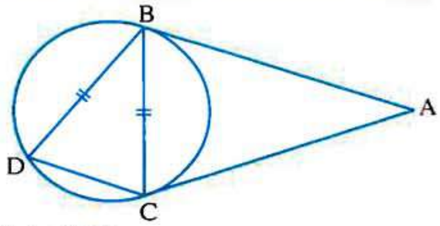


5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , BC = BD

Prove that :

$\overleftrightarrow{BD}$ is a tangent to the circle passing through the vertices of Δ ABC

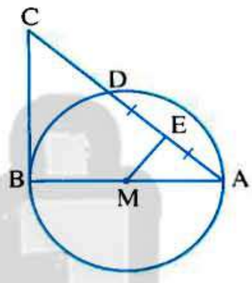


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{BC}$ is a tangent to the circle at B and E is the midpoint of $\overline{AD}$

Prove that : The figure EMBC is a cyclic quadrilateral.



17 El-Menia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1. It is possible to draw a circle passing through the vertices of a ....
   (a) rhombus. (b) rectangle. (c) right trapezium. (d) parallelogram.
2. The inscribed angle drawn in a semicircle is ....
   (a) acute. (b) obtuse. (c) straight. (d) right.
3. The number of rectangles in the opposite figure is ....
   
   (a) 3 (b) 6 (c) 7 (d) 10
4. If the perimeter of a square is 20 cm. , then its surface area is .... cm^2.
   (a) 20 (b) 25 (c) 50 (d) 100
5. The measure of the exterior angle of an equilateral triangle equals .... °
   (a) 60 (b) 108 (c) 120 (d) 135

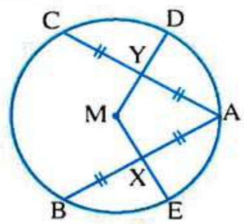
6 If ABCD is a cyclic quadrilateral , 2 m (∠ A) = 120° , then m (∠ C) = ... °

(a) 120 (b) 45 (c) 60 (d) 90

2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$

Prove that : XE = YD



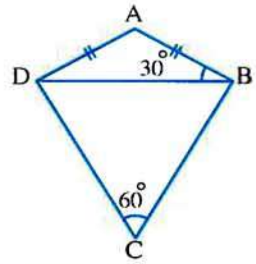
[b] In the opposite figure :

ABCD is a quadrilateral , AB = AD

, m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



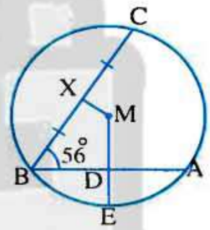
3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M which has radius length of 5 cm. , $\overline{MD} \perp \overline{AB}$

, X is the midpoint of $\overline{BC}$

, AB = 8 cm. , m (∠ B) = 56°

Find : 1 m (∠ DMX) 2 The length of $\overline{DE}$



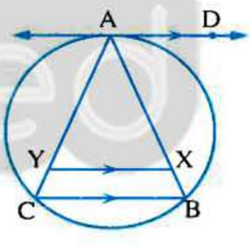
[b] In the opposite figure :

$\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

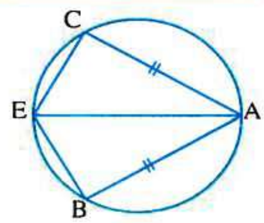


4 **[a] In the opposite figure :**

AB = AC

, E ∈ $\widehat{BC}$

Prove that : m (∠ AEB) = m (∠ AEC)



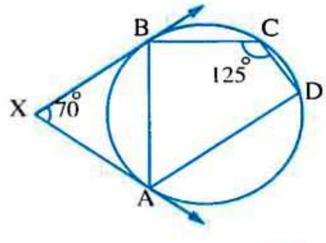
[b] In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 70°

, m (∠ DCB) = 125°

Prove that : m (∠ DAB) = m (∠ XAB)

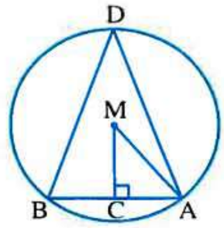


5 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$

Prove that : m ($\angle$ AMC) = m ($\angle$ ADB)



[b] ABC is an inscribed triangle in a circle M where AB > AC and D $\in \overline{AB}$ where AC = AD , $\overrightarrow{AE}$ bisects $\angle$ A and intersects $\overline{BC}$ at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

18 Assiut Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1 XYZ is a triangle in which : D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$, then DE = ... YZ

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

2 The diameter is a .... passing through the center of the circle.

(a) straight line (b) ray (c) tangent (d) chord

3 If the circumference of a circle is 18 π cm. , then its radius length = ... cm.

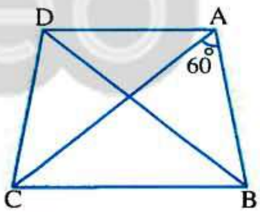
(a) 7 (b) 9 (c) 3 (d) 6

4 **In the opposite figure :**

ABCD is a cyclic quadrilateral

, m ($\angle$ BAC) = 60°

, then m ($\angle$ BDC) = ...



(a) 300° (b) 120°

(c) 60° (d) 30°

5 The area of the triangle which the length of its base is 9 cm. , its height is 12 cm. equals .... cm^2.

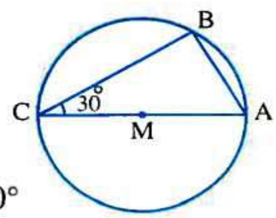
(a) 48 (b) 24 (c) 36 (d) 54

6 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

, m ($\angle$ C) = 30°

, then m ($\angle$ A) = ...



(a) 120° (b) 60° (c) 90° (d) 40°

2 **[a] In the opposite figure :**

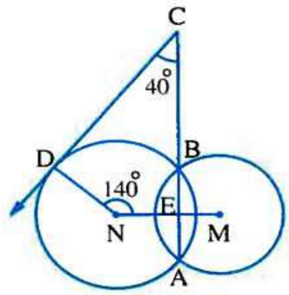
M and N are two intersecting circles at A and B

, C $\in \overrightarrow{AB}$, $\overline{AC} \cap \overline{MN} = \{E\}$

, D $\in$ the circle N , m ($\angle$ DNM) = 140°

and m ($\angle$ C) = 40°

Prove that : $\overrightarrow{CD}$ is a tangent to the circle N at D



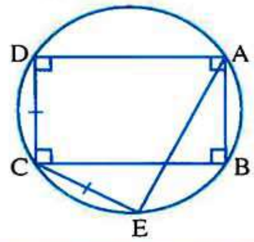
[b] In the opposite figure :

ABCD is a rectangle inscribed in a circle

, the chord $\overline{CE}$ is drawn

where CE = CD

Prove that : AE = BC



3 **[a]** State two cases of the cyclic quadrilateral.

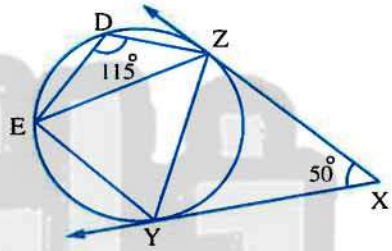
[b] In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, m ($\angle$ D) = 115°

and m ($\angle$ X) = 50°

Prove that : ZE = ZY



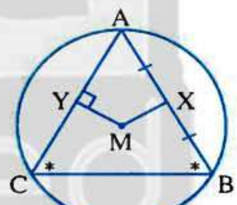
4 **[a] In the opposite figure :**

ABC is a triangle inscribed in the circle M

, in which m ($\angle$ B) = m ($\angle$ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

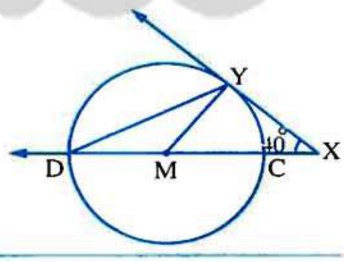
Prove that : MX = MY



[b] In the opposite figure :

X is a point outside the circle M , $\overrightarrow{XY}$ is a tangent to the circle at Y , $\overrightarrow{XM}$ intersects the circle M at C and D respectively , m ($\angle$ X) = 40°

Find : m ($\angle$ YDC)



5 **[a] In the opposite figure :**

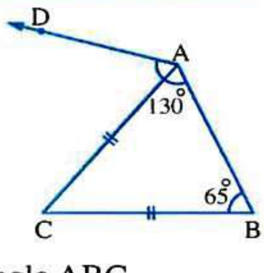
ABC is a triangle , CB = AC

, m ($\angle$ DAB) = 130°

, m ($\angle$ B) = 65°

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



[b] In the opposite figure :

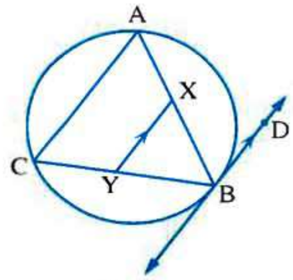
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$

where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



19 Souhag Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer :

1 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the center of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

2 The area of the rhombus = ... of the product of the lengths of its diagonals.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3

3 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

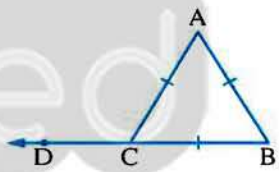
4 **In the opposite figure :**

The triangle ABC is an equilateral triangle

, then m (∠ ACD) = ...°

(a) 45 (b) 60

(c) 120 (d) 135



5 If the lengths of two sides of an isosceles triangle are 2 cm. and $(x + 3)$ cm. , and the length of the third side is 5 cm. , then x = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

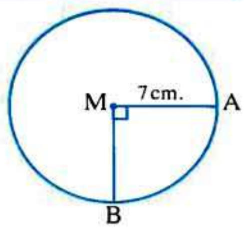
(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

M is a circle with radius length 7 cm.

, m (∠ AMB) = 90°

Find : The length of $\overset{\frown}{AB}$ $\left(\pi = \frac{22}{7}\right)$



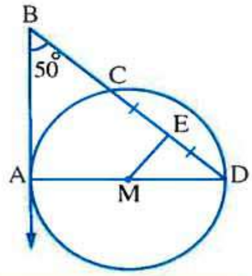
[b] In the opposite figure :

$\overline{AD}$ is a diameter in the circle M

, $\overleftrightarrow{AB}$ is a tangent , m (∠ B) = 50°

, E is the midpoint of $\overline{DC}$

Find : m (∠ EMA)



3 [a] State two cases of the cyclic quadrilateral.

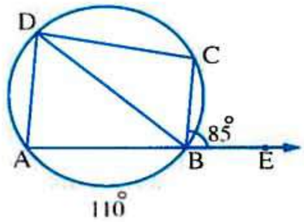
[b] In the opposite figure :

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$

, m ($\widehat{AB}$) = 110°

, m (∠ CBE) = 85°

Find : m (∠ BDC)

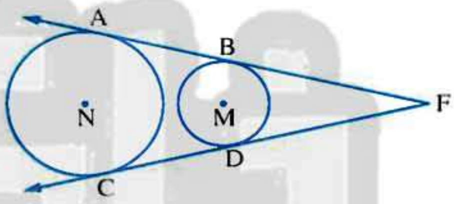


4 **[a] In the opposite figure :**

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are common external tangents

to the two circles M and N , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{F\}$

Prove that : AB = CD



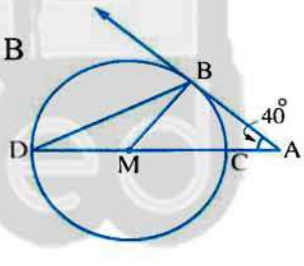
[b] In the opposite figure :

A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B

, $\overrightarrow{AM}$ intersects the circle M at C and D respectively

, m (∠ A) = 40°

Find with proof : m (∠ BDC)

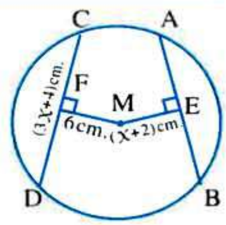


5 **[a] In the opposite figure :**

AB = CD

Find : 1 The value of x

2 The length of $\overline{CD}$



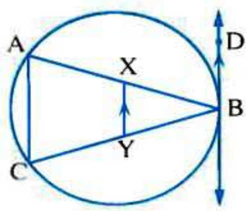
[b] In the opposite figure :

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B , X ∈ $\overline{AB}$

, Y ∈ $\overline{CB}$ where $\overline{YX}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



20 Qena Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer :

1 The measure of the inscribed angle in a semicircle is ·············· °

(a) 45 (b) 90 (c) 135 (d) 180

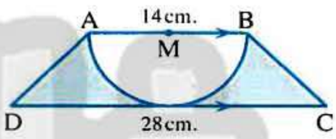
2 The perimeter of a rhombus is 12 cm. , then the length of its side = ·············· cm.

(a) 3 (b) 4 (c) 6 (d) 8

3 If A and B are two points in the plane , AB = 7 cm. , then the length of the diameter of the smallest circle passing through the two points A and B equals ·············· cm.

(a) 3 (b) 3.5 (c) 7 (d) 14

4 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , $\overline{CD}$ is a tangent

, AB = 14 cm. , CD = 28 cm.

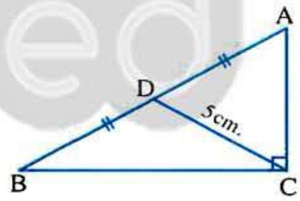
, then the area of the shaded part = ·············· cm^2

(a) 70 (b) 147 (c) 170 (d) 224

5 It is possible to draw a circle passing through the vertices of a ··············

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

6 **In the opposite figure :**



Δ ABC is right-angled at C

, $\overline{CD}$ is a median , CD = 5 cm.

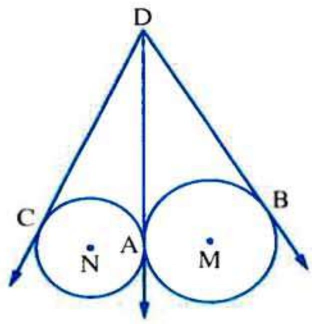
, then AB = ·············· cm.

(a) 4 (b) 6

(c) 8 (d) 10

2 **[a]** Find the length of the arc and its measure , which is opposite to an inscribed angle of measure 45° in a circle the length of its radius is 7 cm.

[b] In the opposite figure :



M and N are two circles touching externally at A

, $\overrightarrow{DA}$ is a common tangent to the circles

, $\overrightarrow{DB}$ is a tangent to the circle M at B

, $\overrightarrow{DC}$ is a tangent to the circle N at C

Prove that : DB = DC

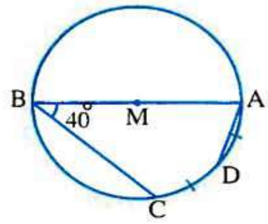
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, D is the midpoint of $\overset{\frown}{AC}$

, m ($\angle$ ABC) = 40°

Find : 1 m ($\angle$ DAB) 2 m ($\angle$ DCB)



[b] In the opposite figure :

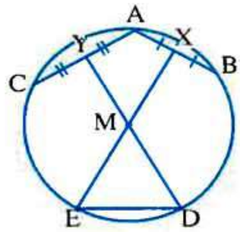
$\overline{AB}$, $\overline{AC}$ are two chords in the circle M

, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overrightarrow{YM}$ and $\overrightarrow{XM}$ intersect the circle at D and E

If DE = r where r is the radius length of M

, find by proof : m ($\angle$ BAC)



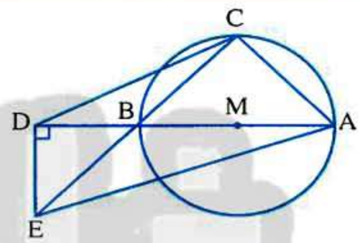
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, D $\in \overrightarrow{AB}$, D $\notin \overline{AB}$, $\overrightarrow{DE} \perp \overleftrightarrow{AB}$

, C $\in \overset{\frown}{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral

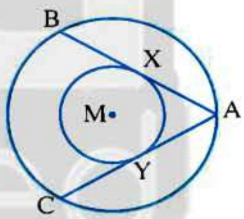


[b] In the opposite figure :

Two concentric circles of center M

, $\overline{AB}$ and $\overline{AC}$ are two chords in the greater circle and tangents to the smaller circle at X and Y respectively.

Prove that : AB = AC

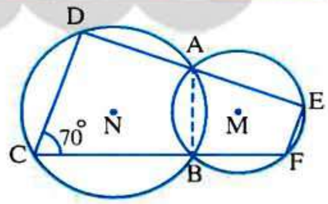


5 **[a] In the opposite figure :**

M and N are two intersecting circles at A and B

, $\overleftrightarrow{AD}$ is drawn to intersect the circle M at E and the circle N at D , $\overleftrightarrow{AB}$ is drawn to intersect the circle M at F and the circle N at C , m ($\angle$ BCD) = 70°

1 **Find :** m ($\angle$ EFB) 2 **Prove that :** $\overline{CD}$ // $\overline{EF}$



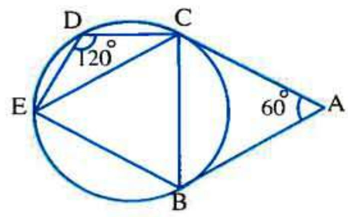
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are tangent-segments to the circle at B and C

, m ($\angle$ BAC) = 60° , m ($\angle$ CDE) = 120°

Prove that : 1 Δ BCE is an equilateral triangle.

2 $\overline{AC}$ // $\overline{BE}$



21 Luxor Governorate

محافظة الأقصر

Answer the following questions :

1 Choose the correct answer :

1 The number of axes of symmetry of the rectangle is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 If M , N are two circles whose radii lengths are r_1 , r_2 and if $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are ....

(a) distant. (b) concentric. (c) intersecting. (d) touching.

3 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) quarter (b) twice (c) half (d) three quarters

4 The length of the arc subtending a central angle of measure 60° in a circle whose circumference is 24 cm. equals .... cm.

(a) 4 (b) 8 (c) 12 (d) 16

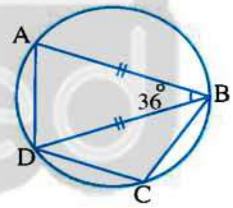
5 The measure of the exterior angle of the equilateral triangle is ....°

(a) 30 (b) 60 (c) 90 (d) 120

6 **In the opposite figure :**

AB = BD , m (∠ ABD) = 36°

, then m (∠ C) = ... °



(a) 140 (b) 108

(c) 70 (d) 54

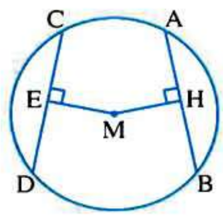
2 [a] **In the opposite figure :**

AB = CD , $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

If ME = 6 cm. , MH = (X + 2) cm.

and CD = (3 X + 4) cm.

, **find :** The value of X and the length of $\overline{AB}$

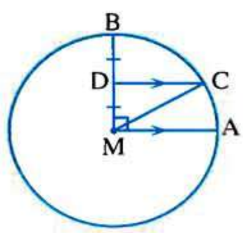


[b] **In the opposite figure :**

$\overline{AM} \parallel \overline{CD}$

, MD = DB , m (∠ AMB) = 90°

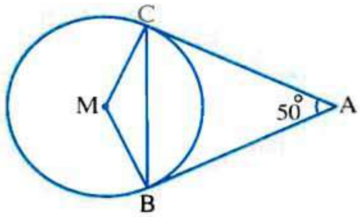
Find : m $(\overset{\frown}{AC})$



3 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent segments drawn to the circle from A at B , C respectively , m (∠ A) = 50°

Find : m (∠ ACB) , m (∠ BCM)



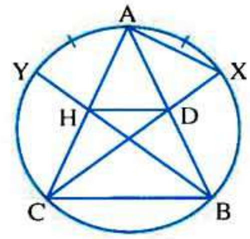
[b] **In the opposite figure :**

m $(\widehat{AX})$ = m $(\widehat{AY})$

Prove that :

1 DBCH is a cyclic quadrilateral.

2 m (∠ DHB) = m (∠ XAB)



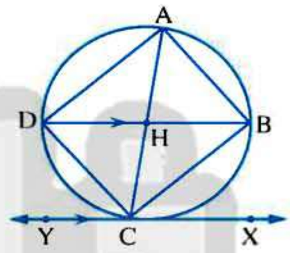
4 [a] Draw $\overline{AB}$ of length 3 cm. , then draw a circle passing by the two points A , B whose radius length is 2 cm. How many possible solutions are there ?

[b] **In the opposite figure :**

$\overline{BD}$ // $\overleftrightarrow{XY}$

Prove that : 1 $\overrightarrow{AC}$ bisects ∠ BAD

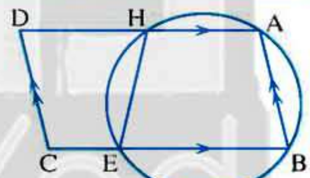
2 $\overleftrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH



5 [a] **In the opposite figure :**

ABCD is a parallelogram

Prove that : HDCE is a cyclic quadrilateral

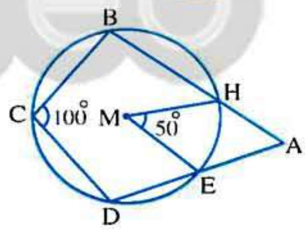


[b] **In the opposite figure :**

m (∠ M) = 50°

, m (∠ C) = 100°

Find : m (∠ A)



22 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 180° (c) 120° (d) 90°

2 The number of symmetry axes of the isosceles triangle is ……………

(a) zero (b) 1 (c) 2 (d) 3

3 The surface of the circle M $\cap$ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle equals …………… cm.

(a) 5 (b) 6 (c) 11 (d) 16

4 The measure of the exterior angle of the equilateral triangle equals ……………

(a) 30° (b) 60° (c) 120° (d) 180°

5 The line segment joining the two midpoints of two sides of the triangle is …………… the third side.

(a) perpendicular to (b) parallel to (c) bisecting (d) equal to

6 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 80° = ……………

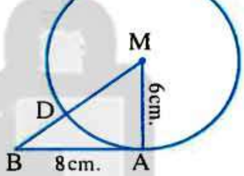
(a) 60° (b) 80° (c) 100° (d) 120°

2 **[a] In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

, MA = 6 cm. , AB = 8 cm.

Find : The length of $\overline{BD}$



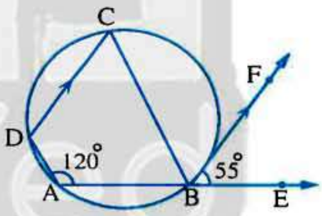
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, $\overrightarrow{BF}$ // $\overline{DC}$, m (∠ BAD) = 120°

, m (∠ EBF) = 55°

Find : m (∠ BCD) , m (∠ ADC)



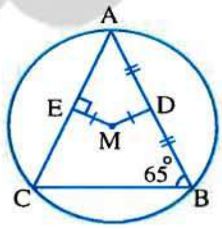
3 **[a] In the opposite figure :**

In the circle M

, MD = ME , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$, m (∠ ABC) = 65°

Find : m (∠ BAC)

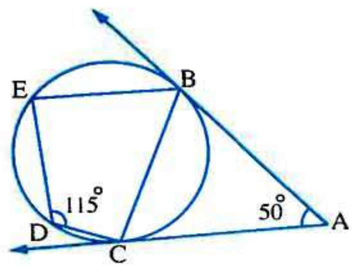


[b] In the opposite figure :

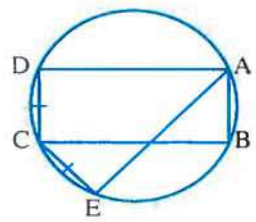
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50° , m (∠ CDE) = 115°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABE



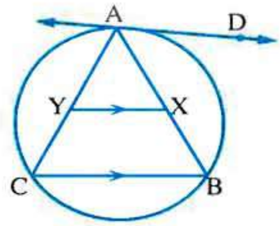
4 **[a] In the opposite figure :**



ABCD is a rectangle inscribed in a circle , the chord $\overline{CE}$ is drawn where CE = CD

Prove that : AE = BC

[b] In the opposite figure :

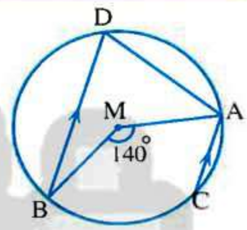


ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , $X \in \overline{AB}$, $Y \in \overline{AC}$, $\overline{XY} \parallel \overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY

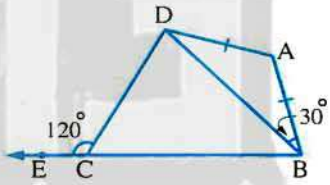
5 **[a] In the opposite figure :**



$\overline{AC}$, $\overline{DB}$ are two parallel chords in the circle M , m ($\angle$ AMB) = 140°

Find : m ($\angle$ D) , m ($\angle$ DAC)

[b] In the opposite figure :



AB = AD , m ($\angle$ ABD) = 30° , m ($\angle$ DCE) = 120°

Prove that : ABCD is a cyclic quadrilateral.

23 New Valley Governorate

إيمان - عمل - تنمية

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm.

(a) 30 (b) 45 (c) 60 (d) 75

2 The inscribed angle drawn in a semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

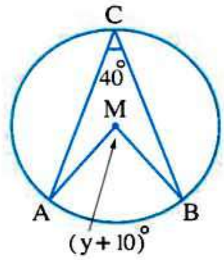
3 ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftarrow{AC}$ is ....

(a) A (b) B (c) C (d) D

4 A tangent to a circle of diameter length 6 cm. is at a distance of .... cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

5 **In the opposite figure :**



If m (∠ AMB) = (y + 10)°

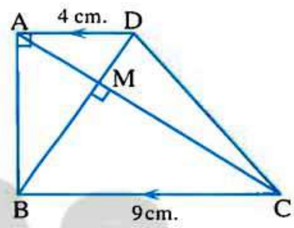
, m (∠ C) = 40°

, then y = ...

(a) 70° (b) 80°

(c) 100° (d) 180°

6 **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$, m (∠ BAD) = m (∠ BMC) = 90°

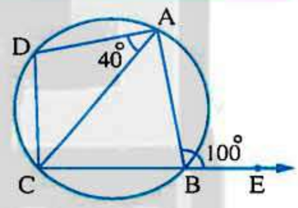
, AD = 4 cm. , BC = 9 cm.

, then the area of the trapezium ABCD = ... cm^2

(a) 26 (b) 39

(c) 52 (d) 65

2 [a] **In the opposite figure :**

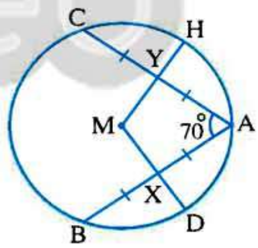


m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$

[b] **In the opposite figure :**



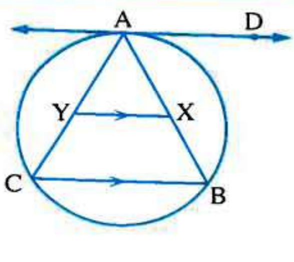
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°

1 **Calculate :** m (∠ DMH)

2 **Prove that :** XD = YH

3 [a] **In the opposite figure :**



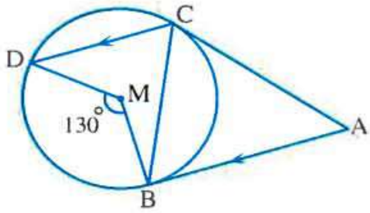
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :



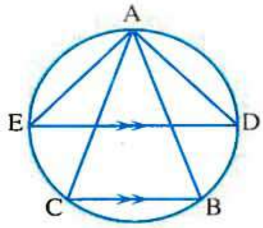
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB} \parallel \overline{CD}$, m (∠ BMD) = 130°

1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

2 **Find :** m (∠ A)

4 **[a] In the opposite figure :**



ABC is an inscribed triangle inside a circle

, $\overline{DE} \parallel \overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

[b] ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$

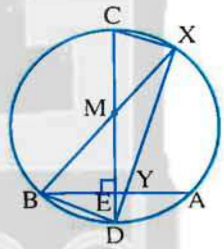
where m $(\widehat{AX})$ = m $(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 BCED is a cyclic quadrilateral.

2 m (∠ DEB) = m (∠ XAB)

5 **[a]** State two cases of the cyclic quadrilateral.

[b] In the opposite figure :



$\overline{AB}$ is a chord in the circle M and $\overline{CD}$ is the perpendicular diameter on $\overline{AB}$ and intersects it at E

, $\overrightarrow{BM}$ intersects the circle at X and $\overline{XD} \cap \overline{AB} = \{Y\}$

Prove that : 1 XYEC is a cyclic quadrilateral.

2 m (∠ DYB) = m (∠ DBX)

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 90° (b) 45° (c) 180° (d) 120°

2 A rhombus whose two diagonals lengths are 6 cm. , 8 cm. , then its area is .... cm^2

(a) 14 (b) 24 (c) 48 (d) 12

3 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 90° = ...

(a) 180° (b) 100° (c) 90° (d) 120°

4 In the triangle ABC , where $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....

(a) right. (b) acute. (c) straight. (d) obtuse.

5 The sum of measures of the interior angles of the triangle equals ....

(a) 180° (b) 90° (c) 100° (d) 360°

6 The number of axes of symmery of the circle is ....

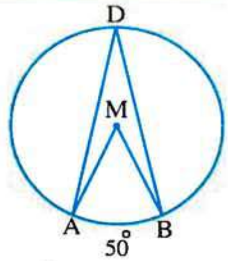
(a) zero (b) an inifinite number

(c) 2 (d) 3

2 [a] **In the opposite figure :**

$m(\widehat{AB}) = 50°$

Find : 1 $m(\angle D)$ 2 $m(\angle AMB)$

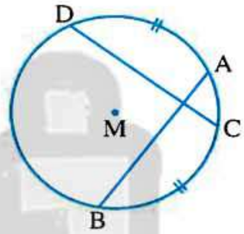


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $m(\widehat{AD}) = m(\widehat{BC})$

Prove that : AB = CD



3 [a] If the radius length of the circle M is 5 cm. and the radius length of the circle N is 3 cm. , MN = 8 cm. , show the position of the two circles.

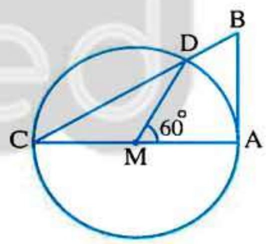
[b] **In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M

, $\overline{AC}$ is a diameter of it and $m(\angle AMD) = 60°$

1 **Find :** $m(\angle ABC)$

2 **Prove that :** $AB = \frac{1}{2}$ BC



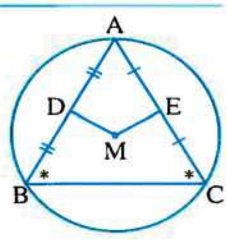
4 [a] **In the opposite figure :**

$m(\angle B) = m(\angle C)$

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

Prove that : MD = ME

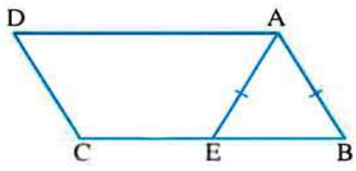


[b] **In the opposite figure :**

ABCD is a parallelogram

and $E \in \overline{BC}$, such that : AB = AE

Prove that : The figure AECD is a cyclic quadrilateral.



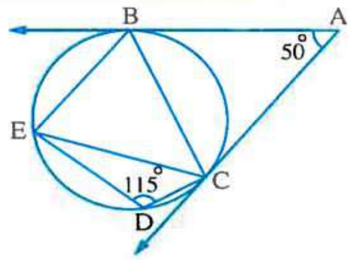
5 **[a] In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 50° , m (∠ EDC) = 115°

Prove that : **1** $\overrightarrow{BC}$ bisects ∠ ABE

2 CB = CE



[b] In the opposite figure :

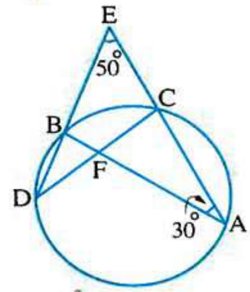
$\overline{AB} \cap \overline{CD} = \{F\}$, $\overrightarrow{AC} \cap \overrightarrow{DB} = \{E\}$

, m (∠ A) = 30°

, m (∠ E) = 50°

Find : **1** m $(\widehat{AD})$

2 m (∠ AFD)



25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If the surface of the circle M ∩ the surface of the circle N = {A}
, then M , N are ……………

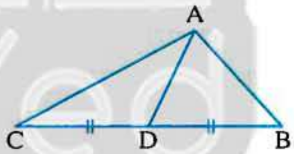
(a) distant. (b) concentric. (c) touching externally. (d) intersecting.

2 **In the opposite figure :**

$\overline{AD}$ is a median in the triangle ABC

, the area of the triangle ABD = 20 cm²

, then the area of the triangle ACD = …………… cm²



(a) 20 (b) 40 (c) 60 (d) 80

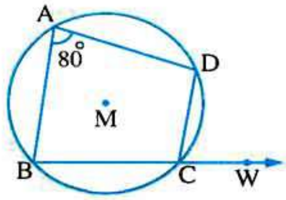
3 **In the opposite figure :**

If m (∠ BAD) = 80°

, then m (∠ DCW) = ……………°

(a) 30 (b) 80

(c) 60 (d) 120



4 The area of the square whose diagonal length is 4 cm. equals …………… cm²

(a) 4 (b) 8 (c) 16 (d) 16 π

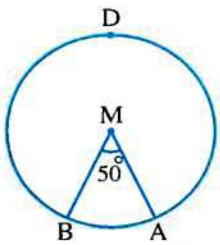
5 **In the opposite figure :**

m (∠ AMB) = 50°

, then m $(\widehat{ADB})$ = ……………°

(a) 50 (b) 100

(c) 310 (d) 350



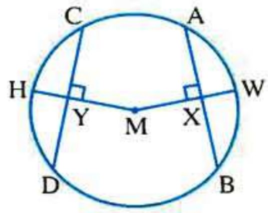
6 A triangle having one symmetry line and its side lengths are 8 , 4 , x cm.
, then x = ...

(a) 2 (b) 4 (c) 8 (d) 12

2 **[a] In the opposite figure :**

If AB = CD
, $\overline{MW} \perp \overline{AB}$
, $\overline{MH} \perp \overline{CD}$

Prove that : WX = HY

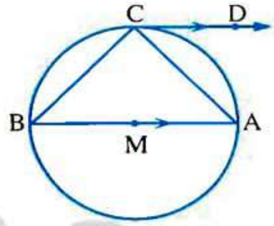


[b] In the opposite figure :

$\overrightarrow{CD}$ is a tangent to the circle M at C
, $\overrightarrow{CD}$ // $\overline{BA}$ and M $\in \overline{AB}$

1 **Prove that :** AC = BC

2 **Find :** m (∠ B)



3 [a] State two cases in which the figure is a cyclic quadrilateral.

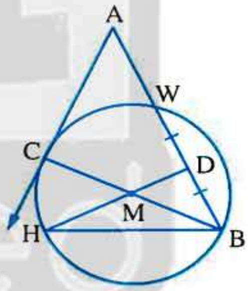
[b] In the opposite figure :

$\overline{BC}$ is a diameter in the circle M
, $\overrightarrow{AC}$ is a tangent to the circle M at C
, D is the midpoint of $\overline{BW}$

Prove that :

1 The figure ADMC is a cyclic quadrilateral.

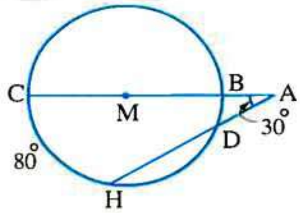
2 m (∠ CBH) = $\frac{1}{2}$ m (∠ A)



4 **[a] In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M
, $\overrightarrow{CA} \cap \overrightarrow{HA} = \{A\}$, m (∠ A) = 30°
and m ($\overset{\frown}{CH}$) = 80°

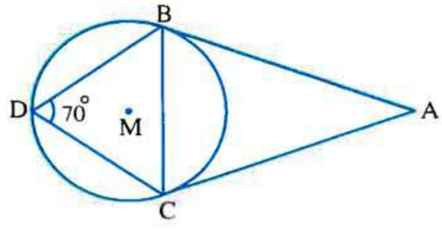
Find : m ($\overset{\frown}{DH}$)



[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B and C
and m (∠ BDC) = 70°

Find : m (∠ BAC)

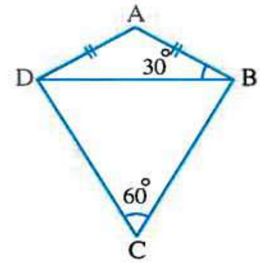


5 **[a] In the opposite figure :**

AB = AD , m (∠ ABD) = 30°

and m (∠ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.



[b] By using geometric instruments , draw Δ ABC where AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then draw a circle passing through the vertices of Δ ABC

How many circles are there ?

26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 The angle of tangency is included between ....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

2 The number of symmetry axes of the semicircle is ....

(a) zero (b) 1 (c) 3 (d) an infinite number.

3 A circle of circumference 6 π cm. and a straight line L is at 3 cm. distant from its centre , then L is ....

(a) a tangent. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

4 The inscribed angle in a semicircle is .... angle.

(a) an acute (b) an obtuse

(c) a straight (d) a right

5 The radius length of the circle whose centre is the point of origin and passes through (– 3 , 4) equals .... length unit.

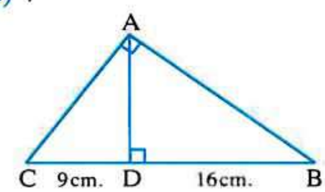
(a) 3 (b) 4 (c) 5 (d) 7

6 **In the opposite figure :**

ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, BD = 16 cm.

, CD = 9 cm. , then AB = ... cm.



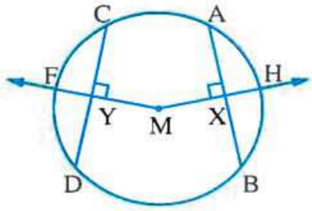
(a) 5 (b) 7 (c) 20 (d) 25

2 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length in the circle M

, $\overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{CD}$

Prove that : HX = FY

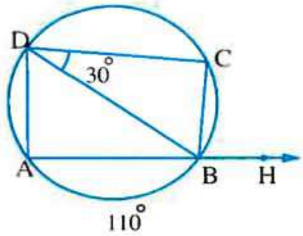


[b] In the opposite figure :

$H \in \overrightarrow{AB}$, m $(\widehat{AB}) = 110°$

, m ($\angle$ CDB) = 30°

Find : m ($\angle$ HBC)

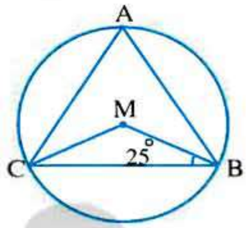


3 **[a] In the opposite figure :**

ABC is a triangle drawn in the circle M

, m ($\angle$ MBC) = 25°

Find : m ($\angle$ BAC)

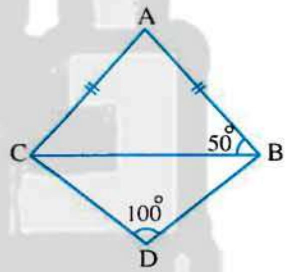


[b] In the opposite figure :

AB = AC , m ($\angle$ D) = 100°

, m ($\angle$ ABC) = 50°

Prove that : ABDC is a cyclic quadrilateral.



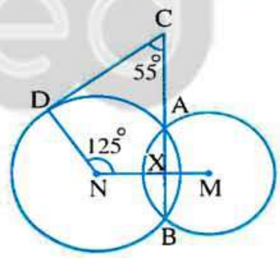
4 **[a] In the opposite figure :**

M and N are two intersecting circles at A and B

, $C \in \overrightarrow{BA}$, D $\in$ the circle N , m ($\angle$ MND) = 125°

, m ($\angle$ C) = 55°

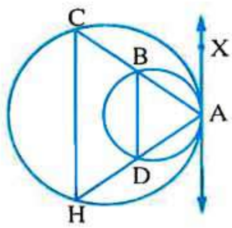
Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



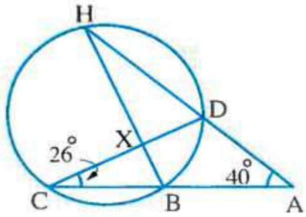
[b] In the opposite figure :

$\overleftrightarrow{AX}$ is a common tangent for the two circles touching internally at A

Prove that : $\overline{BD}$ // $\overline{CH}$



5 **[a] In the opposite figure :**



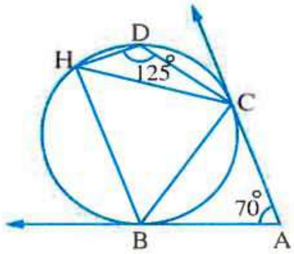
$\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40°

, $\overline{CD} \cap \overline{BH} = \{X\}$

, m (∠ DCB) = 26°

Find : m $(\widehat{CH})$, m (∠ HXC)

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 70°

, m (∠ CDH) = 125°

Prove that : CB = CH

27 Matrouh Governorate

Answer the following questions :

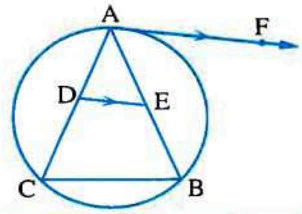
1 **Choose the correct answer from those given :**

1. In the cyclic quadrilateral , each two opposite angles are ....
   (a) equal in measure. (b) complementary.
   (c) supplementary. (d) alternate.
2. A square is of perimeter 20 cm. , then its area equals ....
   (a) 50 cm^2 (b) 50 cm. (c) 25 cm^2 (d) 25 cm.
3. Δ ABC is right-angled at B , if BC = 8 cm. , AB = 6 cm. , then sin C = ...
   (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) 0.6
4. The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc equals ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 1 : 4
5. The measure of the angle of the regular pentagon is equal to ....
   (a) 72° (b) 180° (c) 108° (d) 120°
6. A chord with length 8 cm. in a circle with circumference 10 π cm. , then it is distant from its center by ....
   (a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm.

2 **[a]** $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in a circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , m (∠ MXY) = 30°

Prove that : Δ MXY is an isosceles triangle.

[b] In the opposite figure :

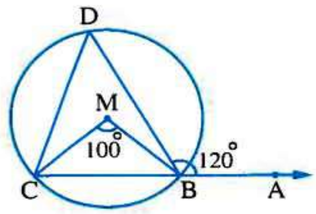


$\overrightarrow{AF}$ is a tangent to the circle at A

, $\overrightarrow{AF}$ // $\overline{DE}$

Prove that : DEBC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**



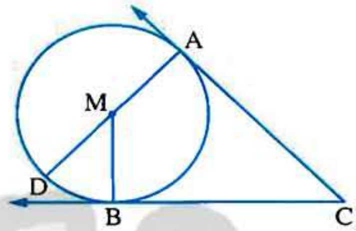
A circle of center M

, m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find : m (∠ DCB)

[b] In the opposite figure :



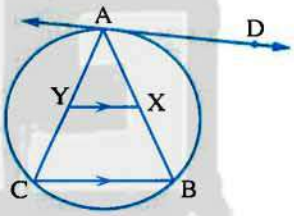
$\overline{AD}$ is a diameter in the circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M

, touching it at A and B respectively.

Prove that : m (∠ DMB) = m (∠ ACB)

4 **[a] In the opposite figure :**



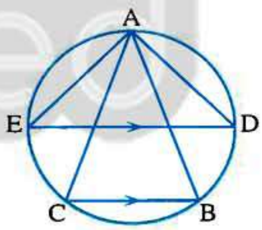
ABC is a triangle inscribed in a circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :



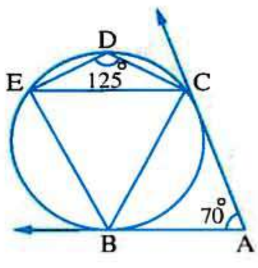
ABC is an inscribed triangle inside a circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

5 **[a] Prove that :** In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

[b] In the opposite figure :



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that : 1 CB = CE

2 $\overrightarrow{AC}$ // $\overline{BE}$

Answers of school book examinations in geometry

Model 1

1

1 d 2 a 3 d 4 a 5 b 6 a

2

[a] supplementary , theoratical.

[b] $\because \overleftrightarrow{XY} // \overrightarrow{BD}$, $\overrightarrow{AB}$ is a transversal

$\therefore m(\angle DBX) = m(\angle BXY)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency) (2)

From (1) and (2) : $\therefore m(\angle C) = m(\angle BXY)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the smaller circle

$\therefore AB = AC$ $\therefore 2x - 3 = 15$

$\therefore 2x = 18$ $\therefore x = 9$

, $\because \overrightarrow{AB}$, $\overrightarrow{AD}$ are two tangents to the greater circle

$\therefore AB = AD$ $\therefore y - 2 = 15$ $\therefore y = 17$

[b] $\because m(\angle BDC) = m(\angle BAC)$

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore m(\angle BDC) = 30°$

$\because m(\widehat{BC}) = 2\,m(\angle BAC) = 60°$

, $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\widehat{AB}) = 180°$

$\therefore m(\widehat{AC}) = 180° - 60° = 120°$

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = \frac{120°}{2} = 60°$ (First req.)

$\therefore m(\angle ACD) = \frac{1}{2}\,m(\widehat{AD}) = \frac{1}{2} \times 60° = 30°$

$\therefore m(\angle CAB) = m(\angle ACD)$ but they are alterante

$\therefore \overline{AB} // \overline{DC}$ (Second req.)

4

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90°$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore (\angle MYA) = 90°$

From the quadrilateral AXMY :

$\therefore m(\angle DMH) = 360° - (90° + 90° + 70°) = 110°$ (First req.)

$\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$, $\because MD = MH = r$

By subtracting : $\therefore XD = YH$ (Second req.)

[b] $\because m(\angle A) = \frac{1}{2}[m(\widehat{HC}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$ $\therefore m(\widehat{BD}) = 120° - 60°$

$\therefore m(\widehat{BD}) = 60°$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{DH})$ $\therefore BC = DH$

by adding $m(\widehat{BD})$ to both sides

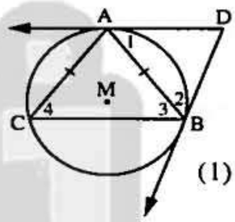
$\therefore m(\widehat{CD}) = m(\widehat{HB})$ $\therefore m(\angle C) = m(\angle H)$

In Δ ACH : $\therefore AC = AH$, $\because BC = DH$

By sbutracting : $\therefore AB = AD$ (Second req.)

5

[a] $\because \overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B



$\therefore DA = DB$

$\therefore m(\angle 1) = m(\angle 2)$

$\therefore m(\angle D)$

$= 180° - 2\,m(\angle 1)$ (1)

In Δ ABC : $\because AB = AC$

$\therefore m(\angle 3) = m(\angle 4)$

$\therefore m(\angle BAC) = 180° - 2\,m(\angle 4)$ (2)

, $\because \overline{AD}$ is a tangent-segment to the circle

$\therefore m(\angle 4)$ (inscribed) $= m(\angle 1)$ (tangency) (3)

From (1) , (2) and (3) : $\therefore m(\angle BAC) = m(\angle D)$

$\therefore \overline{AC}$ is a tangent to the circle passing through the vertices of the Δ ABD (Q.E.D.)

[b] In Δ AMB : $\because AM = BM = r$

$\therefore m(\angle MBA) = m(\angle MAB) = 20°$

$\because$ C is the midpoint of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$ $\therefore m(\angle MCB) = 90°$

In Δ BCM : $\therefore m(\angle BMC) = 180° - (90° + 20°) = 70°$

, $\because m(\angle BHD) = \frac{1}{2}\,m(\angle BMD)$

(inscribed and central angles subtended by $\widehat{BD}$)

$\therefore m(\angle BHD) = \frac{1}{2} \times 70° = 35°$ (First req.)

In Δ AMB : $\because AM = BM = r$

, $m(\angle MAB) = m(\angle MBA) = 20°$

$\therefore m(\angle AMB) = 180° - (20° + 20°) = 140°$

$\therefore m(\widehat{ADB}) = m(< AMB) = 140°$ (Second req.)

Model 2

1

1 b 2 d 3 b
4 c 5 d 6 b

2

[a] ∵ AB = AC

, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ MD = ME , ∵ MX = MY = r

∴ DX = EY (Q.E.D.)

[b] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° – 2 × 30° = 120°

, ∵ m (∠ A) + m (∠ C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

3

[a] State by yourself.

[b] ∵ E is the midpoint of $\overset{\frown}{BF}$

∴ m $(\overset{\frown}{FE})$ = m $(\overset{\frown}{BE})$

∴ m (∠ FAE) = m (∠ BAE)

, ∵ m (∠ CBE) (tangency) = m (∠ BAE) (inscribed)

∴ m (∠ DAC) = m (∠ DBC)

and they are drawn on $\overline{DC}$ and on one side of it

∴ ABCD is a cyclic quadrilateral.

4

[a] ∵ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle

∴ AD = AF = 5 cm.

, ∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

∴ BD = BE = 4 cm.

, ∵ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

∴ CE = CF = 3 cm.

∴ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3
= 24 cm. (The req.)

[b] ∵ $\overleftrightarrow{AF}$ // $\overline{DE}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ AED) = m (∠ EAF) (alternate angles)

, ∵ m (∠ C) (inscribed) = m (∠ BAF) (tangency)

∴ m (∠ C) = m (∠ AED)

∴ DEBC is a cyclic quadrilateral. (Q.E.D.)

5

∵ BCDE is a cyclic quadrilateral

∴ m (∠ CBE) + m (∠ D) = 180°

∴ m (∠ CBE) = 180° – 125° = 55°

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC : m (∠ ACB) = m (∠ ABC)

$= \frac{180° - 70°}{2} = 55°$

∴ m (∠ CBE) = m (∠ ACB) = 55°

and they are alternate angles

∴ $\overleftrightarrow{AC}$ // $\overline{BE}$

, ∵ m (∠ BEC) (inscribed)
= m (∠ ACB) (tangency) = 55°

∴ m (∠ CBE) = m (∠ BEC) = 55°

∴ In Δ CBE : CB = CE

Model examination for the merge students

1

1 diameter 2 perpendicular to this chord
3 equal 4 3 5 infinite
6 180°

2

1 a 2 a 3 d
4 c 5 d 6 c

3

1 ✗ 2 ✓ 3 ✗
4 ✓ 5 ✗ 6 ✗

4

1 90° 2 130° 3 40°
4 5 5 30° 6 2 : 1

Answers of governorates' examinations of geometry

1 Cairo

1

1 c 2 b 3 a 4 a 5 c 6 d

2

[a] Mention by yourself.

[b] $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90^\circ$ (1) (First req.)

, $\because \overline{DE} \perp \overline{AD}$

$\therefore m(\angle ADE) = 90^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle ADE) = m(\angle ACE)$

but they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ The figure ACDE is a cyclic quadrilateral. (Second req.)

3

[a] The measure of the arc $= \frac{1}{3} \times 360^\circ = 120^\circ$ (The req.)

[b] $\because m(\angle BAC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended the same arc $\widehat{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 80^\circ = 40^\circ$

, $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ (First req.)

$\because m(\widehat{BC}) = m(\angle M) = 80^\circ$

$\therefore m(\widehat{BC}$ the major$) = 360^\circ - 80^\circ = 280^\circ$ (Second req.)

4

[a] $\because \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

, $\because$ The sum of measures of the interior angles of the quadrilateral BDME $= 360^\circ$

$\therefore m(\angle DME) = 360^\circ - (70^\circ + 90^\circ + 90^\circ) = 110^\circ$ (First req.)

$\because MD = ME$, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{CB}$

$\therefore AB = CB$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

, $\because \overrightarrow{BD} \mathbin{/\!/} \overrightarrow{AC}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle DBC) = m(\angle ACB)$ (alternate angles) (2)

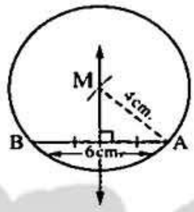
From (1) and (2) :

$\therefore m(\angle ABC) = m(\angle DBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABD$ (Q.E.D.)

5

[a]



$\because$ The radius length of the smallest circle = 3 cm.

[b] $\because \overrightarrow{AD}$ is a tangent to the circle at A

$\therefore m(\angle ABC)$ (inscribed) $= m(\angle CAD)$ (tangency) $= 50^\circ$

, $\because AC = BC$

$\therefore m(\angle BAC) = m(\angle ABC) = 50^\circ$

, $m(\angle BEC) = m(\angle BAC) = 50^\circ$

(two inscribed angles subtended by $\widehat{BC}$) (First req.)

, $\because m(\angle BEC) = m(\angle ABC) = 50^\circ$

$\therefore \overleftrightarrow{BC}$ is a tangent to the circle passing through the vertices of Δ BEO (Second req.)

2 Giza

1

1 d 2 c 3 b 4 b 5 c 6 d

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150^\circ = 75^\circ$

(inscribed and central angles subtened by $\widehat{BD}$)

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle C) = 180^\circ - 75^\circ = 105^\circ$ (The req.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

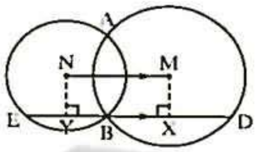
, $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\because$ $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

3

[a] Construction :



Draw $\overline{MX} \perp \overline{BD}$

, $\overline{NY} \perp \overline{BE}$

Proof : $\because$ $\overleftrightarrow{BD} \parallel \overleftrightarrow{MN}$

, $\overline{MX} \perp \overline{BD}$, $\overline{NY} \perp \overline{BE}$

$\therefore$ $\overleftrightarrow{MX} \parallel \overleftrightarrow{NY}$

$\therefore$ The figure MXYN is a rectangle

$\therefore$ X is midpoint of $\overline{BD}$

, Y is midpoint of $\overline{BE}$

$\therefore$ DE = 2 XY , $\because$ XY = MN

$\therefore$ DE = 2 MN (Q.E.D.)

[b] $\because$ $\overleftrightarrow{AB}$ is a tangent to the circle M

$\therefore$ $\overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ MAB) = 90°

In Δ MAB : $\because$ m ($\angle$ ABM) = 30° , m ($\angle$ MAB) = 90°

$\therefore$ BM = 2 AM = 2 × 8 = 16 cm.

$\therefore$ $(AB)^2 = (BM)^2 - (MA)^2 = (16)^2 - (8)^2 = 192$

$\therefore$ AB = $8\sqrt{3}$ cm. (First req.)

, $\because$ AC = $\frac{AM \times AB}{BM}$

$\therefore$ AC = $\frac{8 \times 8\sqrt{3}}{16} = 4\sqrt{3}$ cm. (Second req.)

4

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 50°}{2} = 65°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ EBC) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ EBC) = 180° − 115° = 65°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ EBC)

$\therefore$ $\overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.1)

, $\because$ m ($\angle$ BEC) (inscribed)

= m ($\angle$ ABC) (tangency) = 65°

$\therefore$ m ($\angle$ EBC) = m ($\angle$ BEC)

$\therefore$ In Δ BCE : CB = CE (Q.E.D.2)

[b] $\because$ $\angle$ ABE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD :

$\because$ m ($\angle$ ACD) = 180° − (100° + 40°) = 40°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD)

$\therefore$ CD = AD

$\therefore$ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) (Q.E.D.)

5

[a] $\because$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB) = 60°

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$) (1)

$\because$ $\overleftrightarrow{CD} \parallel \overline{AB}$

$\therefore$ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore$ Δ CAB is an equilateral triangle. (Q.E.D.)

[b] In $\Delta\Delta$ ADE , ACE

$\left\{ \begin{array}{l} \text{m} (\angle \text{DAE}) = \text{m} (\angle \text{CAE}) \\ \text{AD} = \text{AC} \\ \overline{\text{AE}} \text{ is a common side} \end{array} \right.$

$\therefore$ Δ ADE $\equiv$ Δ ACE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ ACE)

, $\because$ m ($\angle$ AFB) = m ($\angle$ ACB)

(two inscribed angles subtended by $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ EFB)

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

3 Alexandria

1

1 b 2 d 3 a 4 b 5 d 6 c

2

[a] $\because$ $\overline{CD}$ is a diameter in a circle M

, AB = 10 cm. , $\overline{MH} \perp \overline{AB}$

$\therefore$ AH = BH = 5 cm.

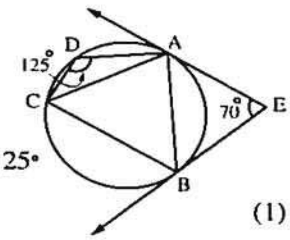
In Δ AHM : $\because$ m ($\angle$ AMH) = 30°

, m ($\angle$ AHM) = 90°

$\therefore$ AM = 2 AH = 10 cm.

$\therefore$ CD = 2 × 10 = 20 cm. (The req.)

[b] $\because$ The figure ABCD is a cyclic quadrilateral.



$\therefore m(\angle ABC) = 180° - 125°$

$= 55°$ (1)

$\because \overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

$\therefore EA = EB$

$\therefore m(\angle EAB) = \frac{180° - 70°}{2} = 55°$

$\because \overrightarrow{EA}$ is a tangent to the circle at A

$\therefore m(\angle EAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) $= 55°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle ABC) = 55°$

$\therefore AB = AC$ (Q.E.D.)

3

[a] $\because m(\angle A) = \frac{1}{2}[m(\widehat{HC}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120° - 60° = 60°$ (First req.)

, $\because m(\widehat{BC}) = m(\widehat{DH})$

$\therefore BC = DH$

By adding $m(\widehat{BD})$ to both sides.

$\therefore m(\widehat{CD}) = m(\widehat{HB})$ $\therefore m(\angle C) = m(\angle H)$

In ΔACH : $\because AC = AH$

, $\because BC = DH$

By subtracting : $AB = AD$ (Second req.)

[b] In ΔABD :

$\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

$\therefore m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB) = 60°$

(inscribed and central angles subtended the same arc $\widehat{AB}$) (1)

$\because \overleftrightarrow{CD} \parallel \overline{AB}$

$\therefore m(\widehat{AC}) = m(\widehat{BC})$

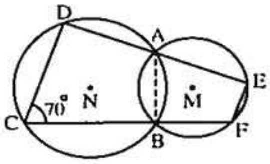
$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta CAB$ is equilateral. (Q.E.D.)

[b] Construction :

Draw $\overline{AB}$



Proof :

$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

, $\because$ The figure ABFE is a cyclic quadrilateral and $\angle BAD$ is an exteroir angle of it

$\therefore m(\angle F) = m(\angle BAD) = 110°$

$\therefore m(\angle F) + m(\angle C) = 110° + 70° = 180°$

but they are two interior angles on the same side of the transversal $\overline{FC}$

$\therefore \overline{CD} \parallel \overline{EF}$ (Q.E.D.)

5

[a] In ΔABC :

$\because AC = BC$

$\therefore m(\angle BAC) = m(\angle ABC) = 65°$

$\therefore m(\angle CAD) = 130° - 65° = 65°$

, $\because m(\angle B) = m(\angle CAD) = 65°$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] $\because \overrightarrow{AD}$ is a tangent

$\therefore \overline{MD} \perp \overline{AD}$ $\therefore m(\angle MDA) = 90°$

, $\because$ H is the midpoint of $\overline{BC}$

$\therefore \overline{MH} \perp \overline{BC}$ $\therefore m(\angle MHA) = 90°$

From the quadrilateral ADMH :

$\therefore m(\angle DMH) = 360° - (56° + 90° + 90°) = 124°$

(The req.)

4 El-Kalyoubia

1

1 c 2 a 3 d 4 b 5 d 6 c

2

[a] $\because \overline{AB} \parallel \overline{CD}$ $\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD}) = 50°$

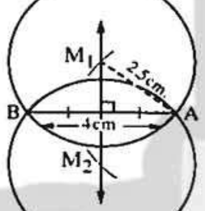
, $\because m(\angle BED) = \frac{1}{2} m(\overset{\frown}{BD})$

$\therefore (3y - 5)° = \frac{1}{2} \times 50° = 25°$

$\therefore 3y = 5° + 25° = 30°$

$\therefore y = 10°$ (The req.)

[b]



$\therefore$ We can draw two circles.

3

[a] $\because$ X is a midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90°$ (1)

, $\because$ Y is a midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle MYA) = 90°$ (2)

From (1) and (2) :

$\therefore m(\angle MXA) = m(\angle MYA)$

but they are drawn on $\overline{AM}$ and on one side of it.

$\therefore$ AXYM is a cyclic quadrilateral. (Q.E.D.1)

In Δ MAC : $\therefore$ MA = MC = r

$\therefore m(\angle MCA) = m(\angle MAC)$

, $\because$ AXYM is a cyclic quadrilateral.

$\therefore m(\angle MXY) = m(\angle MAY)$

$\therefore m(\angle MXY) = m(\angle MCY)$ (Q.E.D.2)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle C) = 180° - 120° = 60°$ (First req.)

$\therefore m(\angle FBC) = m(\angle C) = 60°$ (alternate angles)

$\therefore m(\angle EBC) = 65° + 60° = 125°$

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle D) = m(\angle EBC) = 125°$ (Second req.)

4

[a] $\because$ The circle M $\cap$ The circle N = {A , B}

$\therefore \overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ In Δ ABD :

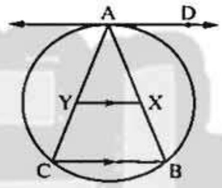
$\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ AD = BD

$\because \overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$

$\therefore$ MX = MY (Q.E.D.)

[b]



$\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

$\therefore$ From (1) and (2) : $\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

5

[a] $\because \overline{AC}$ and $\overline{AB}$ are two tangent-segments to the circle M

$\therefore \overline{AE} \perp \overline{BC}$ $\therefore m(\angle CEM) = 90°$

, $\because \overline{BD}$ is a diameter in the circle M

$\therefore m(\angle ECD) = 90°$

$\therefore m(\angle CEM) + m(\angle ECD) = 180°$

, but they are two interior angles in the same side of the transversal $\overleftrightarrow{BC}$

$\therefore \overline{AM} \parallel \overline{CD}$ (Q.E.D.)

[b] $\because \overline{CM} \parallel \overline{AB}$, $\overleftrightarrow{MA}$ is a transversal.

$\therefore m(\angle MAB) = m(\angle AMC)$ (alternate angles)

, $\because m(\angle AMC) = 2\, m(\angle B)$

(central and inscribed angles subtended by $\overset{\frown}{AC}$)

$\therefore m(\angle EAB) = 2\, m(\angle B)$

$\therefore m(\angle EAB) > m(\angle B)$

From Δ EAB : $\therefore$ BE > AE (Q.E.D.)

5 El-Sharkia

1

1 b   2 a   3 d   4 b   5 d   6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because$ AB = AC $\therefore$ MX = MY

, $\because$ MD = ME = r $\therefore$ XD = YE (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 70°}{2} = 55°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore m(\angle EBC) + m(\angle D) = 180°$

$\therefore m(\angle EBC) = 180° - 125° = 55°$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D.)

3

[a] $\because$ ABDC is a cyclic quadrilateral.

$\therefore m(\angle A) + m(\angle D) = 180°$

$\therefore m(\angle A) = 180° - 140° = 40°$

, $\because \overline{AB}$ is a diameter.

$\therefore m(\angle ACB) = 90°$

In Δ ABC :

$\therefore m(\angle ABC) = 180° - (90° + 40°) = 50°$ (First req.)

, $\because m(\overset{\frown}{BD}) = m(\overset{\frown}{DC})$ $\therefore$ BD = CD

In Δ BCD :

$\therefore m(\angle CBD) = m(\angle BCD) = \frac{180° - 140°}{2} = 20°$

$\therefore m(\overset{\frown}{BD}) = 2\, m(\angle BCD) = 2 \times 20° = 40°$

, $\because m(\overset{\frown}{AB}) = 180°$

$\therefore m(\overset{\frown}{ABD}) = 180° + 40° = 220°$ (Second req.)

[b] $\because \overleftrightarrow{AD}$ is a tangent to the circle

$\because m(\angle DAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$

(corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X and Y (Q.E.D.)

4

[a] $\because m(\overset{\frown}{BD}) = 2\, m(\angle DCB) = 2 \times 25° = 50°$

, $\because$ D is midpoint of $(\overset{\frown}{AB})$

$\therefore m(\overset{\frown}{AB}) = 2 \times 50° = 100°$

$\therefore m(\angle AMB) = m(\overset{\frown}{AB}) = 100°$ (The req.)

[b] $\because \Delta$ ABC is equilateral.

$\therefore m(\angle B) = 60°$

$\therefore m(\angle D) = m(\angle B) = 60°$

(two inscribed angles subtended by $\overset{\frown}{AC}$)

, $\because$ AD = DE

$\therefore \Delta$ ADE is an equilateral triangle. (Q.E.D.1)

$\because m(\angle DAE) = m(\angle BAC) = 60°$

Subtracting $\angle$ BAE from both sides.

$\therefore m(\angle DAB) = m(\angle EAC)$ (Q.E.D.2)

5

[a] $\because \overline{AB}$ is a tangen-tsegment to the circle.

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle A) = 90°$

In Δ MAB : $\because \tan(\angle B) = \frac{AM}{AB}$

$\therefore \tan 30° = \frac{8}{AB}$

$\therefore AB = \frac{8}{\tan 30°} = 8\sqrt{3}$ cm.

In Δ MAB : $\because$ m ($\angle$ AMB) = 180° − (90° + 30°) = 60°

, $\because$ m ($\angle$ XAB) = $\frac{1}{2}$ m ($\angle$ AMB)

(tangency and central angles)

$\therefore$ m ($\angle$ XAB) = $\frac{1}{2}$ × 60° = 30°

In Δ XAB :

$\because$ m ($\angle$ XAB) = m ($\angle$ XBA)

$\therefore$ Δ XAB is an isosceles triangle. (Second req.)

[b] In $\Delta\Delta$ ADE , ACE :

$\begin{cases} m(\angle DAE) = m(\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore$ Δ ADE ≡ Δ ACE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ ACE)

, $\because$ m ($\angle$ AFB) = m ($\angle$ ACB)

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ EFB)

$\therefore$ DBFE is a cyclic quadrilateral. (Q.E.D.)

6 El-Monofia

1

1 c 2 a 3 b 4 b 5 c 6 b

2

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore$ $\overline{AB} \perp \overline{MB}$, $\overline{AC} \perp \overline{MC}$

$\therefore$ m ($\angle$ BAC) = 360° − (90° + 90° + 90°) = 90°

, MB = MC = r

$\therefore$ ABMC is a square. (Q.E.D.)

[b] In Δ AMB : $\because$ AM = MB = r

$\therefore$ m ($\angle$ MAB) = m ($\angle$ ABM)

$\because$ m ($\angle$ CAB) = m ($\angle$ MAB)

$\therefore$ m ($\angle$ CAB) = m ($\angle$ ABM) and they are alternate angles.

$\therefore$ $\overline{AC}$ // $\overline{BM}$

$\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MD} \perp \overline{AC}$

, $\because$ $\overline{AC}$ // $\overline{BM}$

$\therefore$ $\overline{DM} \perp \overline{BM}$ (Q.E.D.)

3

[a] $\because$ $\overline{AX}$, $\overline{AZ}$ are two tangent-segments

$\therefore$ AX = AZ = 6 cm. $\because$ AC = 10 cm.

$\therefore$ CZ = 10 − 6 = 4 cm.

$\because$ $\overline{CY}$, $\overline{CZ}$ are two tangent-segments

$\therefore$ CY = CZ = 4 cm.

$\because$ $\overline{BX}$, $\overline{BY}$ are two tangent-segments

$\therefore$ BX = BY

$\because$ The perimeter of Δ ABC = 24 cm.

$\therefore$ BX + BY + 6 + 10 + 4 = 24

$\therefore$ BX + BY = 4 $\therefore$ BX = 2 cm.

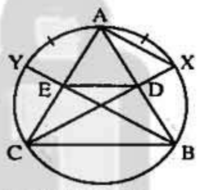
$\therefore$ AB = 6 + 2 = 8 cm. (First req.)

$\because$ $(AC)^2 = (10)^2 = 100$

, $(AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100 = (AC)^2$

$\therefore$ Δ ABC is a right-angled triangle at B (Second req.)

[b] $\because$ m ($\widehat{AX}$) = m ($\widehat{AY}$)



$\therefore$ m ($\angle$ ACX) = m ($\angle$ ABY)

and they are drawn on $\overline{DE}$ and on one side of it

$\therefore$ The figure BCED is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ DCB)

(drawn on $\overline{DB}$ and on one side of it)

, $\because$ m ($\angle$ XAB) = m ($\angle$ XCB)

(two inscribed angles subtended by $\widehat{XB}$)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ XAB) (Q.E.D.2)

4

[a] In Δ ABC : $\because$ CA = CB (1)

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ sin A = sin B

$\therefore$ $\frac{XM}{AM} = \frac{YM}{BM}$ $\because$ AM = BM = r

$\therefore$ XM = YM

, $\because$ $\overline{MX} \perp \overline{DA}$, $\overline{MY} \perp \overline{EB}$

$\therefore$ DA = EB (2)

Subtracting (2) from (1) : $\therefore$ CD = CE (Q.E.D.)

[b] $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ ACB) = 90°

, $\because$ $\overline{ED} \perp \overline{AB}$

$\therefore$ m ($\angle$ FDA) = 90°

$\therefore$ m ($\angle$ ACF) + m ($\angle$ FDA) = 90° + 90° = 180°

$\therefore$ The figure ADFC is a cyclic quadrilateral. (Q.E.D.1)

$\because$ $\overleftrightarrow{EC}$ is a tangent of the circle M

$\therefore$ m ($\angle$ ECB) (tangency) = m ($\angle$ CAB) (inscribed)

, $\because$ $\angle$ CFE is an exterior angle of the cyclic quadrilateral ADFC

$\therefore$ m ($\angle$ CAB) = m ($\angle$ CFE)

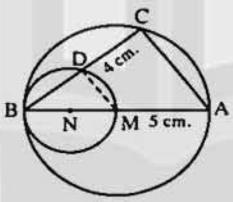
$\therefore$ m ($\angle$ ECF) = m ($\angle$ CFE)

In Δ ECF : $\therefore$ Δ ECF is an isosceles triangle. (Q.E.D.2)

5

[a] Construction :

Draw $\overline{MD}$



Proof :

$\because$ $\overline{BM}$ is a diameter in the circle N

$\therefore$ m ($\angle$ MDB) = 90° $\therefore$ $\overline{MD} \perp \overline{BC}$

$\therefore$ CD = DB = 4 cm.   , $\because$ MB = AM = 5 cm.

In Δ ABC :

$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (10)^2 - (8)^2$
$= 100 - 64 = 36$

$\therefore$ AC = 6 cm. (The req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m ($\angle$ DAB) (tangency)
= m ($\angle$ ACB) (inscribed) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB)
(corresponding angles) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ DAB) = m ($\angle$ AYX)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

7 El-Gharbia

1

1 b	2 a	3 d	4 c	5 b	6 d

2

[a] $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m ($\angle$ ADC) = m ($\angle$ BAD) = 20°
(alternate angles)

, $\because$ m ($\angle$ AEC) = m ($\angle$ ADC) = 20°
(two inscribed angles subtended by $\widehat{AC}$)

$\therefore 3x - 7 = 20$ $\therefore 3x = 27$

$\therefore x = 9$ (The req.)

[b] $\because$ $\overline{BD}$ is a tangent-segment to the circle

$\therefore$ m ($\angle$ ABD) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ MED) = 90°

$\therefore$ m ($\angle$ MBD) + m ($\angle$ MED) = 90° + 90° = 180°

$\therefore$ The figure MEDB ia a cyclic quadrilateral. (Q.E.D.1)

$\because$ $\angle$ BMX is an exterior angle of the cyclic quadrilateral MEDB

$\therefore$ m ($\angle$ D) = m ($\angle$ BMX)

, $\because$ m ($\angle$ BAX) = $\frac{1}{2}$ m ($\angle$ BMX)
(inscribed and central angles subtended by $\widehat{XB}$)

$\therefore$ m ($\angle$ BAX) = $\frac{1}{2}$ m ($\angle$ D) (Q.E.D.2)

3

[a] In Δ ABC : $\because$ m ($\angle$ BAC) = 90°

$\therefore \tan B = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore$ m ($\angle$ B) = 30°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ DAC) = 30°

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments of the circle

$\therefore$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

, $\because$ $\overline{AB}$ // $\overline{CD}$ and $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ BCD) = m ($\angle$ ABC) (2)
(alternate angles)

From (1) and (2) : $\therefore$ m ($\angle$ BCD) = m ($\angle$ ACB)

$\therefore$ $\overrightarrow{CB}$ bisects $\angle$ ACD (Q.E.D.)

4

[a] $\because \angle AMB$ is an exterior angle of the ΔAMD

$\therefore m(\angle AMB) = m(\angle ADM) + m(\angle DAM)$

$\therefore 80^\circ = 30^\circ + m(\angle DAM)$

$\therefore m(\angle DAM) = 80^\circ - 30^\circ = 50^\circ$

In ΔADC : $\because DA = DC$

$\therefore m(\angle DCA) = m(\angle DAC) = 50^\circ$

$\therefore m(\angle ABD) = m(\angle ACD)$

and they are drawn on $\overline{AD}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ X is the midpont of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = ME = r$

$\therefore XD = YE$ (Q.E.D)

5

[a] $\because m(\widehat{AD}) = 2\, m(\angle ABD) = 2 \times 22^\circ = 44^\circ$

, $\because m(\angle C) = \frac{1}{2}[m(\widehat{BE}) - m(\widehat{AD})]$

$\because 36^\circ = \frac{1}{2}[m(\widehat{BE}) - 44^\circ]$

$\therefore 72^\circ = m(\widehat{BE}) - 44^\circ$

$\therefore m(\widehat{BE}) = 116^\circ$ (The req.)

[b] $\because m(\angle BDC) = m(\angle BAC)$

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore m(\angle BDC) = 30^\circ$ (First req.)

$\because m(\widehat{BC}) = 2\, m(\angle BAC) = 60^\circ$

, $\because \overline{AB}$ is diameter in the circle M

$\therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{AC}) = 180^\circ - 60^\circ = 120^\circ$

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = \frac{120^\circ}{2} = 60^\circ$

$\therefore m(\angle ACD) = \frac{1}{2}\, m(\widehat{AD}) = \frac{1}{2} \times 60^\circ = 30^\circ$

$\therefore m(\angle BAC) = m(\angle ACD)$ but they are alternate angles

$\therefore \overline{DC} \parallel \overline{AB}$ (Second req.)

8 El-Dakahlia

1

[a] 1 a 2 d 3 c

[b] $\because \angle ABH$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle ADC) = m(\angle ABH) = 110^\circ$

In ΔACD :

$\because m(\angle ACD) = 180^\circ - (110^\circ + 35^\circ) = 35^\circ$

$\therefore m(\angle CAD) = m(\angle ACD)$ $\therefore CD = AD$

$\therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

2

[a] 1 c 2 a 3 d

[b] $\because \overrightarrow{AB}, \overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$ (First req.)

$\because$ BCHD is a cyclic quadrilateral

$\therefore m(\angle C) + m(\angle D) = 180^\circ$

$\therefore m(\angle C) = 180^\circ - 125^\circ = 55^\circ$

$\because m(\angle BHC)$ (inscribed)

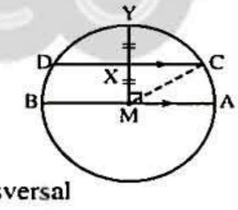
$= m(\angle ABC)$ (tangency) $= 55^\circ$

$\therefore m(\angle BCH) = m(\angle BHC)$

In ΔBCH : $\therefore CB = BH$ (Second req.)

3

[a] Construction :



Draw $\overline{MC}$

Proof :

$\because \overline{CD} \parallel \overline{AB}$, $\overrightarrow{MY}$ is a transversal

$\therefore m(\angle MXC) + m(\angle XMA) = 180^\circ$

$\therefore m(\angle MXC) = 90^\circ$

, $\because MX = \frac{1}{2} MY$, $MY = MC$

$\therefore MX = \frac{1}{2} MC$ $\therefore m(\angle MCX) = 30^\circ$

$\therefore m(\angle AMC) = m(\angle MCX) = 30^\circ$ (alternate angles)

$\therefore m(\widehat{AC}) = m(\angle AMC) = 30^\circ$ (First req.)

, $\because (\widehat{AY}) = m(\angle AMY) = 90^\circ$

$\therefore m(\widehat{CY}) = 90^\circ - 30^\circ = 60^\circ$ (Second req.)

[b] $\because AB = AC$

, $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore MD = MH$

, $\because MX = MY = r$ $\qquad \therefore XD = HY$ (Q.E.D.)

4

[a] $\because \overline{AO} \parallel \overline{DH}$, $\overleftrightarrow{AH}$ is a transversal

$\therefore m(\angle HAO) = m(\angle AHD)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed)

$= m(\angle BAO)$ (tangency) (2)

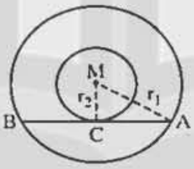
From (1) and (2) :

$\therefore m(\angle C) = m(\angle AHD)$

$\therefore$ DHBC is a cyclic quadrilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MA}$, $\overline{MC}$



Proof :

$\because \overline{AB}$ touches the smaller circle at C

$\therefore \overline{MC} \perp \overline{AB}$

, $\because \overline{AB}$ is a chord of the greater circle

, $\overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = \frac{14}{2} = 7$ cm.

$\because \Delta AMC$ is a right-angled at C

$\therefore (AC)^2 = (MA)^2 - (MC)^2$

$\therefore (7)^2 = r_1 - r_2$ $\qquad \therefore r_1 - r_2 = 49$

$\therefore$ The area of the part included between the two circles = The area of the greater circle − The area of the smaller circle $= \pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$

$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$. (The req.)

5

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended the same arc $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

$\because \overline{CD} \parallel \overline{AB}$ $\qquad \therefore m(\widehat{AC}) = m(\widehat{BC})$

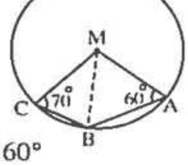
$\therefore AC = BC$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is equilateral (Q.E.D.)

[b] **Construction :**

Draw $\overline{MB}$



Proof :

In ΔMAB :

$\because MA = MB = r$, $m(\angle MAB) = 60°$

$\therefore \Delta AMB$ is equilateral

$\therefore m(\angle AMB) = 60°$ (1)

In ΔMBC : $\because MB = MC = r$

$\therefore m(\angle MBC) = m(\angle MCB) = 70°$

$\therefore m(\angle CMB) = 180° - (70° + 70°) = 40°$ (2)

From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle AMB) + m(\angle CMB)$

$= 60° + 40° = 100°$ (The req.)

9 Ismailia

1

1 c   2 b   3 c   4 a   5 d   6 b

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMC) = x°$

(inscribed and central angles subtended by $\widehat{BC}$)

, $\because$ The figure ABDC is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle BDC) = 180°$

$\therefore x + 2x = 180°$ $\qquad \therefore 3x = 180°$

$\therefore x = 60°$ $\qquad \therefore m(\angle A) = 60°$ (The req.)

[b] $\because m(\angle A) = m(\angle B)$

(two inscribed angles subtended by $\widehat{CD}$)

, $m(\angle C) = m(\angle D)$

(two inscribed angles subtended by $\widehat{AB}$)

$\because EA = ED$ $\qquad \therefore m(\angle A) = m(\angle D)$

$\therefore m(\angle C) = m(\angle B)$

$\therefore EB = EC$ (Q.E.D.)

3

[a] In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because \overrightarrow{BX}$ bisects $(\angle ABC)$, $\overrightarrow{CY}$ bisects $(\angle ACB)$

$\therefore m(\angle XBY) = m(\angle YCX)$

and they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ BCXY is a cyclic quadrilateral (Q.E.D.)

[b] $\because$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\widehat{BC}$)

$\therefore$ m ($\angle$ BAC) = $\frac{1}{2} \times 120° = 60°$

In Δ ABC : $\therefore$ m ($\angle$ C) = 180° – (70° + 60°) = 50°

$\therefore$ m ($\angle$ DAB) = m ($\angle$ C) = 50°

(inscribed and tangency angles subtended by $\widehat{AB}$)

(The req.)

4

[a] $\because$ $\overline{AC}$ is a diameter of the circle.

$\therefore$ m ($\angle$ ABC) = 90°

, $\because$ m ($\angle$ ABD) = 60°

$\therefore$ m ($\angle$ CBD) = 90° – 60° = 30° (First req.)

, $\because$ m ($\angle$ ADB) = m ($\angle$ C) = 50°

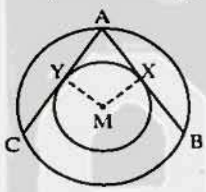
(two inscribed angles subtended by $\widehat{AB}$)

In Δ ABD :

$\therefore$ m ($\angle$ BAD) = 180° – (60° + 50°) = 70°

(Second req.)

[b] Construction :



Draw $\overline{MX}$, $\overline{MY}$

Proof :

In the smaller circle M

$\because$ $\overline{AB}$, $\overline{AC}$ are two tangents

, $\overline{MX}$, $\overline{MY}$ are two radii

$\therefore$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

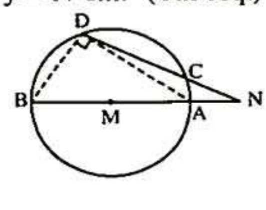
$\therefore$ $2x - 3 = 15$ $\therefore$ $2x = 18$

$\therefore$ $x = 9$ cm.

, $\because$ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

$\therefore$ y – 2 = 15 $\therefore$ y = 17 cm. (The req.)

[b] Construction :



Draw $\overline{AD}$, $\overline{BD}$

Proof :

$\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ m ($\angle$ ADB) + m ($\angle$ ADN) > 90°

In Δ NDB : $\therefore$ NB > ND (Q.E.D.)

10 Suez

1

1 b	2 b	3 a	4 c	5 d	6 b

2

[a] $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$

, $\because$ $\overline{MD} \perp \overline{AB}$, MD = ME

$\therefore$ AB = AC (Q.E.D.)

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore$ m ($\angle$ A) = $\frac{1}{2} \times 100° = 50°$ (First req.)

In Δ MBC : $\because$ MB = MC = r

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB)

$= \frac{1}{2}$ (180° – 100°) = 40°

(Second req.)

3

[a] $\because$ $\overline{AB}$ is a diameter of the circle

$\therefore$ m ($\angle$ AEB) = 90° (First req.)

, $\because$ $\angle$ AEB is an exterior angle of Δ AEC

$\therefore$ m ($\angle$ AEB) = m ($\angle$ CAE) + m ($\angle$ ACE)

$\therefore$ m ($\angle$ CAE) = 90° – 60° = 30° (Second req.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ $\overline{MD} \perp \overline{AD}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$ $\therefore$ m ($\angle$ MEA) = 90°

$\therefore$ In the quadrilateral ADME :

$\therefore$ m ($\angle$ DME) = 360° – (90° + 90° + 70°) = 110°

(The req.)

4

[a] State by yourself.

[b] $\because$ ABC is an equilateral triangle

$\therefore m(\angle A) = 60°$

$\therefore m(\angle D) = m(\angle A)$ and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $m(\widehat{AB}) = 2\, m(\angle ADB) = 60°$ (First req.)

, $m(\angle DCB) = \frac{1}{2}[m(\widehat{AD}) + m(\widehat{AB})]$

$= \frac{1}{2}[90° + 60°] = 75°$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{1}{2}(180° - 40°) = 70°$ (First req.)

$\because \overrightarrow{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle BCD) = m(\angle ABC) = 70°$ (1)

(alternate angles)

, $\because m(\angle BDC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 70°$ (2)

From (1) and (2) :

$\therefore m(\angle BCD) = m(\angle BDC)$

$\therefore$ In Δ BCD : BC = BD (Second req.)

11 Port Said

1

1 d 2 c 3 b 4 b 5 a 6 b

2

[a] $\because$ MF = ME (lengths of two radii)

, XF = YE $\therefore$ MX = MY

$\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore$ AB = CD (Q.E.D.1)

$\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$, $\because \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoinf of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$, $\because$ AB = CD

$\therefore$ AX = CY

$\therefore$ In $\Delta\Delta$ AXF , CYE

$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90° \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$, AF = CE (Q.E.D.2)

[b] $\because m(\angle A) = \frac{1}{2}[m(\widehat{CE}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120° - 60° = 60°$ (The req.)

3

[a] In Δ ABC : $\because m(\angle BAC) = 90°$, $AC = \frac{1}{2} BC$

$\therefore m(\angle B) = 30°$

$\therefore m(\angle C) = 180° - (90° + 30°) = 60°$

$\therefore m(\angle C) = m(\angle DAB) = 60°$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC (Q.E.D.)

[b] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90°$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle AEM) = 90°$

From the quadrilateral MDAE :

$\therefore m(\angle DME) = 360° - (90° + 90° + 120°) = 60°$

, $\because m(\angle YMX) = m(\angle DME) = 60°$ (V.O.A)

, MY = MX = r

$\therefore \Delta$ XMY is an equilateral triangle. (Q.E.D.)

4

[a] In Δ AMC : $\because$ MA = MC = r

$\therefore m(\angle MCA) = m(\angle MAC) = 25°$ (1)

In Δ BMC : $\because$ MB = MC = r

$\therefore m(\angle MCB) = m(\angle MBC) = 45°$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle MCA) + m(\angle MCB)$

$\therefore m(\angle ACB) = 25° + 45° = 70°$

$\therefore m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 70° = 140°$

(central and inscribed angles subtended by $\widehat{AB}$)

(The req.)

[b] $\because$ ABCE is a cyclic quadrilateral

$\therefore m(\angle XEA) = m(\angle ABC)$

, $\because$ ABDF is a cyclic quadrilateral

$\therefore m(\angle XFA) = m(\angle ABD)$

, $\because m(\angle ABC) + m(\angle ABD) = 180^\circ$

$\therefore m(\angle XEA) + m(\angle XFA) = 180^\circ$

$\therefore$ AFXE is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the greater circle

$\therefore AB = AC$

$\therefore 2x - 3 = 15$ $\therefore 2x = 18$

$\therefore x = 9$ cm.

, $\because \overline{AC}$, $\overline{AD}$ are two tangent-segments to the smaller circle

$\therefore AC = AD$ $\therefore y - 2 = 15$

$\therefore y = 17$ cm. (The req.)

[b] $\because$ ABCD is a parallelogram

$\therefore AD = BC$, $\because BE = AD$

$\therefore BC = BE$

$\therefore$ In Δ BCE : $m(\angle C) = m(\angle BEC)$

, $\because m(\angle C) = m(\angle BAD)$ (from the parallelogram)

$\therefore m(\angle BAD) = m(\angle BED)$ and they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

12 Damietta

1

1 b   2 d   3 c   4 b   5 a   6 b

2

[a] $\because \overleftrightarrow{AD}$ is a tangent

$\therefore \overline{MD} \perp \overline{AD}$ $\therefore m(\angle MDA) = 90^\circ$

, $\because$ E is a midpoint of $\overline{BC}$

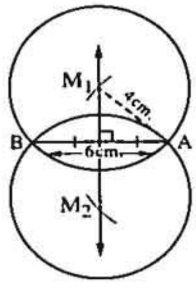
$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90^\circ$

From the quadrilateral ADME

$\therefore m(\angle DME) = 360^\circ - (65^\circ + 90^\circ + 90^\circ) = 115^\circ$

(The req.)

[b]



$\therefore$ We can draw two circles.

3

[a] $\because m(\angle BMC) = 2\, m(\angle A)$

(central and inscribed angles subtended by $\widehat{BC}$)

$\therefore m(\angle BMC) = 2 \times 30^\circ = 60^\circ$ (First req.)

In Δ MBC : $\because MB = MC = r$

, $m(\angle BMC) = 60^\circ$

$\therefore \Delta$ MBC is equilateral. (Second req.)

[b] $\because \overline{AD} \mathbin{/\!/} \overline{BC}$

$\therefore m(\widehat{AB}) = m(\widehat{DC})$ $\therefore AB = DC$

, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{DC}$

$\therefore MX = MY$ (Q.E.D.)

4

[a] $\because \overrightarrow{CB}$ is a tangent

$\therefore m(\angle BAE) = m(\angle CBE)$

(inscribed and tangency angles subtended by $\widehat{BE}$)

, $\because m(\widehat{BE}) = m(\widehat{EA})$

$\therefore m(\angle BAE) = m(\angle EAF)$

$\therefore m(\angle CBD) = m(\angle CAD)$ and they are drawn on $\overline{CD}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

[b] $\because m(\angle XYZ)$ (tangency) $= m(\angle L)$ (inscribed) $= 70^\circ$

, $\because \overline{XY}$, $\overline{XZ}$ are two tangents

$\therefore XY = XZ$

$\therefore m(\angle XYZ) = m(\angle XZY) = 70^\circ$

In Δ XYZ :

$\therefore m(\angle X) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (First req.)

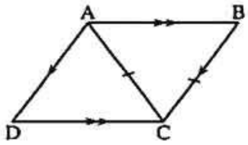
In Δ LZY : $\because YZ = LZ$

$\therefore m(\angle LYZ) = m(\angle L) = 70^\circ$

, $\because m(\angle LYZ) = m(\angle XZY)$ and they are alternate angles.

$\therefore \overline{XZ} \mathbin{/\!/} \overline{YL}$ (Second req.)

5



[a] In ABC :

$\because$ AC = BC

$\therefore$ m ($\angle$ B) = m ($\angle$ CAB) (1)

, $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AC}$ is transversal

$\therefore$ m ($\angle$ DCA) = m ($\angle$ CAB) (alternate angles) (2)

From (1) and (2) : $\therefore$ m ($\angle$ DCA) = m ($\angle$ B)

$\therefore$ $\overrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC (Q.E.D.)

[b] $\because$ LMNE is a cyclic quadrilateral

$\therefore$ m ($\angle$ MLN) = m ($\angle$ MEN) = 35° (First req.)

, $\because$ m ($\angle$ ELN) = m ($\angle$ ELM) – m ($\angle$ MLN)

$\therefore$ m ($\angle$ ELN) = 80° – 35° = 45°

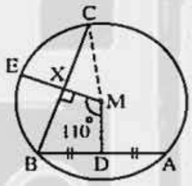
$\therefore$ m ($\angle$ EMN) = m ($\angle$ ELN) = 45° (Second req.)

13 Kafr El-Sheikh

1

1 a 2 c 3 c 4 b 5 b 6 d

2



[a] **Construction :**

Draw $\overline{MC}$

Proof :

$\because$ $\overline{MX} \perp \overline{BC}$

$\therefore$ X is the midpoint of $\overline{BC}$

$\therefore$ XC = 8 cm.

In Δ XMC :

$\because$ m ($\angle$ CXM) = 90° , CM = r = 10 cm.

$\therefore$ MX = $\sqrt{(CM)^2 - (XC)^2} = \sqrt{100 - 64}$

$= \sqrt{36} = 6$ cm.

$\therefore$ XE = 10 – 6 = 4 cm. (First req.)

$\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$

From the quadrilateral BDMX :

$\therefore$ m ($\angle$ ABC) = 360° – (90° + 90° + 110°) = 70° (Second req.)

[b] $\because$ $\overrightarrow{BA}$ is a tangent

$\therefore$ $\overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ BAM) = 90°

In ΔAMB : m ($\angle$AMB) = 180° – (90° + 20°) = 70°

, $\because$ m ($\angle$ ADE) = $\frac{1}{2}$ m ($\angle$ AME)

(inscribed and central angles subtended by $\widehat{AE}$)

$\therefore$ m ($\angle$ ADB) = $\frac{1}{2}$ × 70° = 35° (The req.)

3

[a] $\because$ $\overline{AD}$ // $\overline{CB}$ $\therefore$ m ($\widehat{BD}$) = m ($\widehat{AC}$)

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CDA)

$\therefore$ In Δ ADE : EA = ED (Q.E.D.)

[b] $\because$ $\overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle

$\therefore$ EA = EB

In Δ ABE :

$\therefore$ m ($\angle$ EAB) = m ($\angle$ EBA) = $\frac{180° - 50°}{2}$ = 65°

, $\because$ m ($\angle$ ADC) (inscribed)

= m ($\angle$ CAE) (tangency) = 115°

$\therefore$ m ($\angle$ BAC) = 115° – 65° = 50°

$\because$ m ($\angle$ AEB) = m ($\angle$ BAC)

$\therefore$ $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ ADC) = m ($\angle$ ABE) = 110°

, $\because$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\widehat{AB}$) = $\frac{1}{2}$ × 100° = 50°

$\therefore$ m ($\angle$ BDC) = 110° – 50° = 60° (The req.)

[b] $\because$ $\overrightarrow{FB}$, $\overrightarrow{FD}$ are two tangents to the circle

$\therefore$ BF = DF = 4 cm.

$\therefore$ AB = 10 + 4 = 14 cm.

, $\because$ $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two tangents to the circle

$\therefore$ AC = AB = 14 cm.

$\therefore$ EC = 14 – 9 = 5 cm. (The req.)

5

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

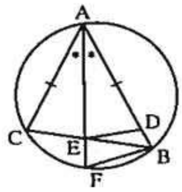
$\therefore$ $\overline{MY} \perp \overline{AC}$, $\because$ MX = MY

$\therefore$ AB = AC

In Δ ABC : $\therefore$ m ($\angle$ C) = m ($\angle$ B) = 70°

$\therefore$ m ($\angle$ A) = 180° – (70° + 70°) = 40° (The req.)

[b] In $\Delta\Delta$ ADE , ACE



$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$

, $m(\angle ADE) = m(\angle ACE)$

, $\because m(\angle AFB) = m(\angle ACB)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle AFB) = m(\angle ADE)$

$\therefore$ BDEF is a cyclic quadrilateral. (Q.E.D.)

14 El-Beheira

1

1 d　2 c　3 b　4 b　5 c　6 a

2

[a] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$　　$\therefore m(\angle AXY) = 90°$

, $\because \overleftrightarrow{YB}$ is a tangent to the circle

$\therefore \overline{MB} \perp \overleftrightarrow{BY}$　　$\therefore m(\angle MBY) = 90°$

, $\because m(\angle AXY) = m(\angle ABY)$ and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overline{CM} \mathbin{/\mkern-6mu/} \overline{AB}$, $\overleftrightarrow{AM}$ is a transversal

$\therefore m(\angle CMA) = m(\angle A) = 60°$

, $\because m(\angle B) = \frac{1}{2}\, m(\angle CMA)$

(two inscribed angles subtended by $\widehat{AC}$)

$\therefore m(\angle B) = \frac{1}{2} \times 60° = 30°$ (The req.)

3

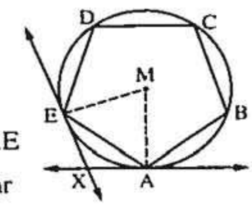
[a] $\because m(\angle B) = m(\angle C)$　　$\therefore AB = AC$

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] Construction :



Draw $\overline{AM}$, $\overline{ME}$

Proof :

$\because AB = BC = CD = DE = AE$

(The properties of the regular pentagon)

$\therefore m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{CD}) = m(\widehat{DE}) = m(\widehat{AE})$

$\because$ measure of the circle $= 360°$

$\therefore m(\widehat{AE}) = \frac{360°}{5} = 72°$ (First req.)

$\therefore m(\angle AME) = m(\widehat{AE}) = 72°$

$\because \overleftrightarrow{AX}$ is a tangent to the circle at A

$\therefore m(\angle MAX) = 90°$

similarly $m(\angle MEX) = 90°$

In the quadrilateral MAXE :

$\therefore m(\angle AXE) = 360° - (72° + 90° + 90°) = 108°$

(Second req.)

4

[a] In Δ AMC : $\because AM = MC = r$

$\therefore m(\angle MAC) = m(\angle ACM)$

$\because m(\angle BAC) = m(\angle MAC)$

$\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles.

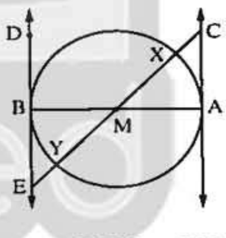
$\therefore \overline{AB} \mathbin{/\mkern-6mu/} \overline{CM}$

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$　　, $\because \overline{AB} \mathbin{/\mkern-6mu/} \overline{CM}$

$\therefore \overline{DM} \perp \overline{CM}$ (Q.E.D.)

[b] $\because \overleftrightarrow{AC}$ is a tangent to the circle M at A



$\therefore \overline{MA} \perp \overleftrightarrow{AC}$

$\therefore m(\angle CAM) = 90°$

$\because \overleftrightarrow{BD}$ is a tangent to the circle M at B

$\therefore \overleftrightarrow{MB} \perp \overleftrightarrow{BD}$　　$\therefore m(\angle EBM) = 90°$

In $\Delta\Delta$ CAM , EBM :

$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90° \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$

$\therefore$ The two triangles are congruent and we deduce that CM = EM

, $\because$ XM = YM (lengths of two radii)

, by subtracting

$\therefore$ CX = YE (Q.E.D.)

5

[a] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

$\therefore$ In Δ ABX

$m(\angle XAB) = m(\angle XBA) = \frac{180° - 50}{2} = 65°$

, $\because$ ABCD is a cyclic quadrilateral

$m(\angle BAD) + m(\angle DCB) = 180°$

$\therefore m(\angle BAD) = 180° - 115° = 65°$

$\therefore m(\angle XAB) = m(\angle BAD)$

$\therefore \overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.1)

, $\because m(\angle ADB)$ (inscribed)

$= m(\angle XAB)$ (tangency) $= 65°$

$\therefore m(\angle BAD) = m(\angle ADB)$

$\therefore$ BD = BA (Q.E.D.2)

[b] $\because$ AB = CD

$\therefore m(\widehat{AB}) = m(\widehat{CD})$

Subtracting $m(\widehat{BD})$ from both sides

$\therefore m(\widehat{AD}) = m(\widehat{BC})$

$\therefore m(\angle ACD) = m(\angle BAC)$

$\therefore$ In Δ ACE : AE = CE

$\therefore \Delta$ ACE is an isosceles triangle. (Q.E.D.)

15 El-Fayoum

1

1 c 2 b 3 a 4 d 5 a 6 c

2

[a] $\because$ AB = CD

, $\overline{ME} \perp \overline{AB}$, $\overline{MO} \perp \overline{CD}$

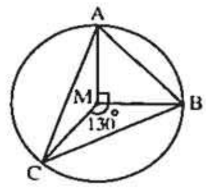
$\therefore$ ME = MO $\therefore x + 2 = 6$

$\therefore x = 4$ cm. (First req.)

$\therefore$ CD = AB = $3 \times 4 + 4 = 16$ cm. (Second req.)

[b] $\because m(\angle C) = \frac{1}{2} m(\angle AMB)$

$= \frac{1}{2} \times 90° = 45°$



(inscribed and central angles subtended by $\widehat{AB}$)

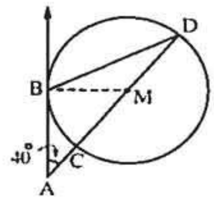
, $\because m(\angle A) = \frac{1}{2}(\angle BMC) = \frac{1}{2} \times 130° = 65°$

(inscribed and central angles subtended by $\widehat{BC}$)

$\therefore m(\angle B) = 180° - (45° + 65°) = 70°$ (The req.)

3

[a] Constraction :



Draw $\overline{MB}$

Proof :

$\because \overleftrightarrow{AB}$ is a tangent to the circle

$\therefore \overline{MB} \perp \overleftrightarrow{AB}$

$\therefore m(\angle MBA) = 90°$

In Δ ABM :

$m(\angle BMA) = 180° - (90° + 40°) = 50°$

$m(\angle BDC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 50° = 25°$

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\therefore m(\angle AXY) = 90°$

, $\because \overleftrightarrow{YB}$ is a tangent to the circle

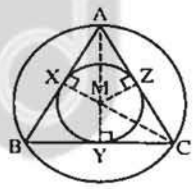
$\therefore \overline{MB} \perp \overline{BY}$ $\therefore m(\angle MBY) = 90°$

, $\because m(\angle AXY) = m(\angle ABY)$ and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilateral (Q.E.D.)

4

[a] Construction :



Draw $\overline{XM}$, $\overline{YM}$, $\overline{ZM}$, $\overline{AY}$, $\overline{CM}$

Proof :

$\because \overline{XM} \perp \overline{AB}$, $\overline{YM} \perp \overline{BC}$, $\overline{ZM} \perp \overline{AC}$

, $\because$ XM = YM = ZM = r

$\therefore$ AB = BC = AC

$\therefore \Delta$ ABC is an equilateral triangle (First req.)

In Δ MYC : $m(\angle MYC) = 90°$

$\therefore (YC)^2 = (MC)^2 - (MY)^2 = (4)^2 - (2)^2 = 12$

$\therefore YC = 2\sqrt{3}$ cm. $\therefore BC = 4\sqrt{3}$ cm.

$\therefore$ The area of Δ ABC $= \frac{1}{2} \times BC \times AY$

$= \frac{1}{2} \times 4\sqrt{3} \times 6$

$= 12\sqrt{3}$ cm^2. (Second req.)

[b] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ BCD) = $\frac{1}{2} \times 130° = 65°$

$\because$ $\overline{AB}$ // $\overline{CD}$, $\overrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65°

(alternate angles) (1)

, $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent segments

$\therefore$ AB = AC

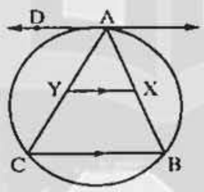
$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = 65° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ACB) = m ($\angle$ BCD) = 65°

$\therefore$ $\overrightarrow{CB}$ bisects $\angle$ ACD (Q.E.D.)

5



[a] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m ($\angle$ DAC) (tangency)

= m ($\angle$ B) (inscribed) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$

is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (2)

(corresponding angles)

From (1) and (2) : $\therefore$ m ($\angle$ AXY) = m ($\angle$ DAC)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MX} \perp \overline{AC}$

$\therefore$ m ($\angle$ CXM) = 90°

, $\because$ $\overleftrightarrow{BD}$ is a tangent to the circle

$\therefore$ $\overline{BD} \perp \overline{AB}$

$\therefore$ m ($\angle$ DBM) = 90°

$\therefore$ m ($\angle$ CXM) + m ($\angle$ DBM) = 180°

$\therefore$ XMBD is a cyclic quadrilateral (Q.E.D.1)

, $\because$ $\angle$ BMY is an exterior angle of the cyclic quadrilateral XMBD

$\therefore$ m ($\angle$ BMY) = m ($\angle$ D) (1)

, $\because$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ BMY) (2)

(inscribed and central angles subtended the same arc $\overset{\frown}{BY}$)

From (1) and (2) :

$\therefore$ m ($\angle$ BAY) = $\frac{1}{2}$ m ($\angle$ D) (Q.E.D.2)

16 Beni Suef

1

1 c 2 a 3 c 4 c 5 b 6 c

2

[a] $\because$ m ($\angle$ AMB) = 2 m ($\angle$ ADB) = 2 × 70° = 140°

(central and inscribed angles subtended by $\overset{\frown}{AB}$)

In Δ ABM : $\because$ $\overline{MC} \perp \overline{AB}$

, MA = MB = r

$\therefore$ m ($\angle$ AMC) = $\frac{1}{2}$ m ($\angle$ AMB) = $\frac{1}{2} \times 140 = 70$

(The req.)

[b] $\because$ AB = CD

, $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$

$\therefore$ MX = NY , $\overline{MX}$ // $\overline{NY}$

$\therefore$ MXYN is a rectangle (Q.E.D.)

3

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$ $\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{ME} \perp \overline{AC}$ $\therefore$ m ($\angle$ AEM) = 90°

$\therefore$ ADME is a cyclic quadrilateral

$\therefore$ m ($\angle$ DME) = 360° − (90° + 90° + 50°) = 130°

(The req.)

[b] $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACB) = 55°

, $\because$ m ($\angle$ BDC) = m ($\angle$ BAC) 55° and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

4

[a] $\because$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB)

(inscribed and central angles subtended the same arc $\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ACB) = $\frac{1}{2} \times 120° = 60°$ (1)

$\because$ $\overleftrightarrow{ED}$ // $\overline{AB}$

$\therefore$ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

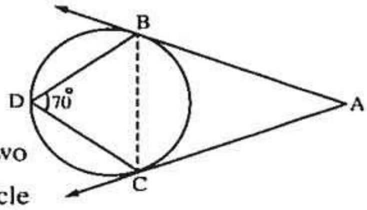
$\therefore$ AC = BC (2)

From (1) and (2) :

$\therefore$ Δ CAB is an equilateral triangle. (Q.E.D.)

[b] Construction :

Draw $\overline{BC}$



Proof :

$\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because m(\angle ABC)$ (tangency) $= m(\angle BDC)$ (inscribed) $= 70°$

$\therefore$ In ΔABC :

$m(\angle A) = 180° - (70° + 70°) = 40°$ (The req.)

5

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$ (1)

In ΔBCD : $\because BC = BD$

$\therefore m(\angle BDC) = m(\angle BCD)$ (2)

, $\because m(\angle BDC)$ (inscribed) $= m(\angle ABC)$ (tangency) (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) = m(\angle CBD)$

$\therefore \overrightarrow{BD}$ is a tangent to the circle passing through the vertices of ΔABC (Q.E.D.)

[b] $\because \overleftrightarrow{BC}$ is a tangent to the circle

$\therefore \overline{AB} \perp \overline{BC}$

$\therefore m(\angle ABC) = 90°$

, $\because$ E is the midpoint of $\overline{AD}$

$\therefore \overline{ME} \perp \overline{AD}$

$\therefore m(\angle CEM) = 90°$

$\therefore m(\angle ABC) + m(\angle CEM) = 180°$

$\therefore$ EMBC is a cyclic quadrilateral (Q.E.D.)

17 El-Menia

1

1 b 2 d 3 b 4 b 5 c 6 a

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$, $\because AB = AC$

$\therefore MX = MY$, $\because ME = MD = r$

$\therefore XE = YD$ (Q.E.D.)

[b] In ΔABC : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

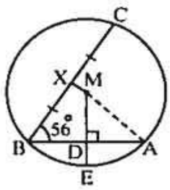
, $\because m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

3

[a] Construction :

Draw $\overline{AM}$



Proof :

$\because \overline{MD} \perp \overline{AB}$

$\therefore m(\angle MDB) = 90°$

, $\because$ X is the midpoint of $\overline{BC}$

$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle MXB) = 90°$

In the quadrilateral MDXB :

$\therefore m(\angle DMX) = 360° - (56° + 90° + 90°) = 124°$ (First req.)

$\because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = 4$ cm.

In ΔADM :

$(MD)^2 = (AM)^2 - (AD)^2 = (5)^2 - (4)^2 = 25 - 16 = 9$

$\therefore MD = 3$ cm.

$\therefore DE = 5 - 3 = 2$ cm. (Second req.)

[b] $\because \overrightarrow{AD}$ is a tangent to the circle

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} // \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

$\therefore$ From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] $\because AB = AC$

$\therefore m(\widehat{AB}) = m(\widehat{AC})$

$\therefore m(\angle AEB) = m(\angle AEC)$ (Q.E.D.)

[b] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore m(\angle XAB) = m(\angle XBA) = \frac{180° - 70}{2} = 55°$ (1)

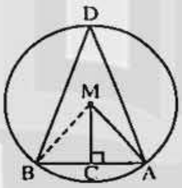
, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle DAB) = 180° - 125° = 55°$ (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle XAB)$ (Q.E.D.)

5



[a] Construction :

Draw $\overline{MB}$

Proof :

$\because MA = MB = r$

, $\overline{MC} \perp \overline{AB}$

$\therefore \overrightarrow{MC}$ bisects $\angle AMB$

$\therefore m(\angle AMC) = \frac{1}{2} m(\angle AMB)$ (1)

, $\because m(\angle ADB) = \frac{1}{2} m(\angle AMB)$ (2)

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore$ From (1) and (2) :

$\therefore m(\angle AMC) = m(\angle ADB)$ (Q.E.D.)

[b] $\because$ In $\Delta\Delta$ ADE , ACE

$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore \Delta ADE \equiv \Delta ACE$

$\therefore m(\angle ADE) = m(\angle ACE)$

, $\because m(\angle AFB) = m(\angle ACB)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle AFB) = m(\angle ADE)$

$\therefore$ BDEF is a cyclic quadrilateral. (Q.E.D.)

18 Assiut

1

1 c 2 d 3 b 4 c 5 d 6 b

2

[a] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord.

$\therefore \overline{AB} \perp \overline{MN}$ $\therefore m(\angle BEN) = 90°$

In the quadrilateral CDNE :

$\therefore m(\angle CDN) = 360° - (140° + 40° + 90°) = 90°$

$\therefore \overline{ND} \perp \overline{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because AB = CD$ (properties of the rectangle)

, $\because CE = CD$ $\therefore AB = CE$

$\therefore m(\widehat{AB}) = m(\widehat{CE})$ and adding $m(\widehat{BE})$ to both sides.

$\therefore m(\widehat{AE}) = m(\widehat{BC})$

$\therefore AE = BC$ (Q.E.D.)

3

[a] State by yourself.

[b] $\because \overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle

$\therefore XY = XZ$

$\therefore$ In ΔXYZ :

$m(\angle XYZ) = m(\angle XZY) = \frac{180° - 50°}{2} = 65°$

, $\because$ YZDE is a cyclic quadrilateral

$\therefore m(\angle EYZ) + m(\angle D) = 180°$

$\therefore m(\angle EYZ) = 180° - 115° = 65°$

, $\because m(\angle YEZ)$ (inscribed)

$= m(\angle XYZ)$ (tangency) $= 65°$

$\therefore m(\angle EYZ) = m(\angle YEZ)$

$\therefore$ In $\Delta YZE : ZE = ZY$ (Q.E.D.)

4

[a] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because \overleftrightarrow{XY}$ is a tangent to the circle

$\therefore \overline{MY} \perp \overline{XY}$ $\therefore m(\angle XYM) = 90°$

In ΔXYM :

$\therefore m(\angle XMY) = 180° - (90° + 40°) = 50°$

, ∵ m (∠ YDC) = $\frac{1}{2}$ m (∠ YMC)

(inscribed and central angles subtended by $\widehat{YC}$)

∴ m (∠ YDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] In Δ ABC : ∵ CB = AC

∴ m (∠ BAC) = m (∠ ABC) = 65°

∴ m (∠ CAD) = 130° − 65° = 65°

, ∵ m (∠ ABC) = m (∠ CAD) = 65°

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ YXB) (alternate angles) (1)

, ∵ m (∠ C) (inscribed) = m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ YXB)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

19 Souhag

1

1 b 2 c 3 d 4 c 5 b 6 b

2

[a] ∵ m (∠ AMB) = 90° ∴ m ($\widehat{AB}$) = 90°

, ∵ r = 7 cm.

∴ The length of $\widehat{AB}$ = $\frac{90°}{360°}$ × 2 × $\frac{22}{7}$ × 7 = 11 cm. (The req.)

[b] ∵ $\overrightarrow{AB}$ is a tangent

∴ $\overline{MA}$ ⊥ $\overrightarrow{AB}$ ∴ m (∠ MAB) = 90°

, ∵ E is the midpoint of $\overline{DC}$

∴ $\overline{ME}$ ⊥ $\overline{DC}$ ∴ m (∠ MEB) = 90°

From the quadrilateral ABEM :

∴ m (∠ EMA) = 360° − (50° + 90° + 90°) = 130° (The req.)

3

[a] State by yourself.

[b] ∵ ∠ CBE is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ ADC) = m (∠ CBE) = 85°

, ∵ m (∠ ADB) (inscribed) = $\frac{1}{2}$ m ($\widehat{AB}$)

= $\frac{1}{2}$ × 110° = 55°

∴ m (∠ BDC) = 85° − 55° = 30° (The req.)

4

[a] ∵ $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are two tangents to the circles M , N

In circle M

BF = DF (1)

, in circle N : AF = CF (2)

Subtracting (1) from (2) :

∴ AF − BF = CF − DF

∴ AB = CD (Q.E.D.)

[b] ∵ $\overleftrightarrow{AB}$ is a tangent to the circle

∴ $\overline{MB}$ ⊥ $\overleftrightarrow{AB}$ ∴ m (∠ ABM) = 90°

In Δ ABM :

∴ m (∠ AMB) = 180° − (40° + 90°) = 50°

, ∵ m (∠ BDC) = $\frac{1}{2}$ m (∠ BMC)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ BDC) = $\frac{1}{2}$ × 50° = 25° (The req.)

5

[a] ∵ AB = CD , $\overline{ME}$ ⊥ $\overline{AB}$, $\overline{MF}$ ⊥ $\overline{CD}$

∴ ME = MF ∴ x + 2 = 6

∴ x = 4 cm. (First req.)

, $\overline{CD}$ = 3 × 4 + 4 = 16 cm. (Second req.)

[b] ∵ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ BXY) (alternate angles) (1)

, ∵ m (∠ C) (inscribed) = m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ BXY)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

20 Qena

1

1 b 2 a 3 c 4 a 5 b 6 d

2

[a] The measure of the arc $= 45° \times 2 = 90°$

, its length $= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 7$

$= 11$ cm. (The req.)

[b] $\because \overrightarrow{DB}$, $\overrightarrow{DA}$ are two tangent to the circle M

$\therefore$ DB = DA (1)

, $\because \overrightarrow{DC}$, $\overrightarrow{DA}$ are two tangent to the circle N

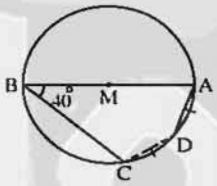
$\therefore$ DC = DA (2)

From (1) and (2) : $\therefore$ DB = DC (Q.E.D.)

3

[a] Construction :

Draw $\overline{CD}$



Proof :

$\because$ D is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{DC}) = 40°$

$\because \overline{AB}$ is a diameter

$\therefore m(\overset{\frown}{BC}) = 180° - (40° + 40°) = 100°$

$\therefore m(\angle DAB) = \frac{1}{2} m(\overset{\frown}{BD}) = \frac{1}{2}(100° + 40°)$

$= \frac{1}{2} \times 140°$

$= 70°$ (First req.)

, $\because m(\angle DCB) = \frac{1}{2} m(\overset{\frown}{BAD}) = \frac{1}{2}(180° + 40°)$

$= \frac{1}{2} \times 220° = 110°$

(Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two chords in the circle.

, X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore m(\angle MXA) = 90°$, $m(\angle MYA) = 90°$

In Δ MDE : $\because$ DE = MD = ME = r

$\therefore m(\angle EMD) = 60°$

, $\because m(\angle XMY) = m(\angle EMD) = 60°$ (V.O.A.)

In the quadrilateral AXMY :

$\therefore m(\angle BAC) = 360° - (90° + 90° + 60°) = 120°$

(The req.)

4

[a] $\because \overline{AB}$ is a diameter of the circle.

$\therefore m(\angle ACB) = 90°$

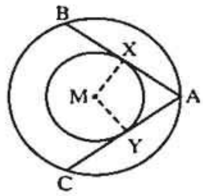
$\therefore m(\angle ACE) = m(\angle ADE)$

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral. (Q.E.D.)

[b] Construction :

Draw $\overline{MX}$, $\overline{MY}$



Proof :

$\because \overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle.

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r (radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

5

[a] $\because$ ABCD is a cyclic quadr quadrilateral ilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

, $\because$ ABFE is a cyclic quadrilateral and $\angle$ BAD is exterior of it.

$\therefore m(\angle EFB) = m(\angle BAD) = 110°$ (First req.)

$\therefore m(\angle EFB) + m(\angle BCD) = 110° + 70° = 180°$

and they are interior angle in the same side of $\overleftrightarrow{FC}$

$\therefore \overline{CD} \,//\, \overline{EF}$ (Second req.)

[b] $\because \overline{AB}$, $\overline{AC}$ are tangent-segments to the circle

$\therefore$ AB = AC

$\therefore m(\angle ACB) = \frac{180° - 60}{2} = 60°$ (1)

$\therefore m(\angle BEC)$ (inscribed)

$= m(\angle ACB)$ (tangency) $= 60°$ (2)

, $\because$ EBCD is cyclic quadrilateral

$\therefore m(\angle EBC) = 180° - 120° = 60°$ (3)

$\therefore$ From (2) , (3) in Δ EBC :

$\therefore m(\angle BCE) = 60°$

$\therefore \Delta$ BCE is equilateral (Q.E.D. 1)

From (1) , (3) : $\therefore m(\angle ACB) = m(\angle EBC)$ and they are alternate angles

$\therefore \overline{AC} \,//\, \overline{BE}$ (Q.E.D. 2)

21 Luxor

1

1 b 2 c 3 c 4 a 5 d 6 b

2

[a] ∵ AB = CD

, $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

∴ MH = ME ∴ $x + 2 = 6$

∴ $x = 4$ cm. (First req.)

∴ AB = CD = 3 × 4 + 4 = 16 cm. (Second req.)

[b] ∵ $\overrightarrow{AM} // \overrightarrow{CD}$, $\overleftrightarrow{MD}$ is a transversal.

∴ m (∠ CDM) + m (∠ AMD) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ CDM) = 180° – 90° = 90°

, ∵ MD = $\frac{1}{2}$ MB , MC = MB = r

∴ MD = $\frac{1}{2}$ MC ∴ m (∠ MCD) = 30°

, ∵ $\overrightarrow{AM} // \overrightarrow{CD}$, $\overleftrightarrow{CM}$ is a transversal.

∴ m (∠ AMC) = m (∠ MCD) = 30°

(alternate angles)

∴ m ($\widehat{AC}$) = m (∠ AMC) = 30° (The req.)

3

[a] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent segments

∴ AB = AC

∴ m (∠ ACB) = m (∠ ABC) = $\frac{180° - 50°}{2}$ = 65°

(First req.)

, ∵ $\overline{MC}$ is a radius ∴ $\overline{MC} \perp \overline{AC}$

∴ m (∠ ACM) = 90°

∴ m (∠ BCM) = 90° – 65° = 25° (Second req.)

[b] ∵ m ($\widehat{AX}$) = m ($\widehat{AY}$)

∴ m (∠ ACX) = m (∠ ABY)

, ∵ They are drawn on $\overline{HD}$ and on one side of it.

∴ DBCH is a cyclic quadrilateral. (Q.E.D.1)

∴ m (∠ DHB) = m (∠ DCB)

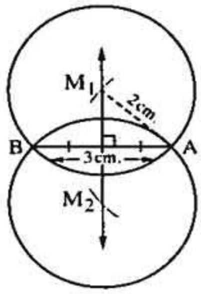
, ∵ m (∠ XCB) = m (∠ XAB)

(two inscribed angles subtended by $\widehat{XB}$)

∴ m (∠ DHB) = m (∠ XAB) (Q.E.D.2)

4

[a]



∴ There are two solutions.

[b] ∵ $\overline{BD} // \overline{XY}$ ∴ m ($\widehat{BC}$) = m ($\widehat{CD}$)

∴ m (∠ BAC) = m (∠ DAC) (1)

∴ $\overrightarrow{AC}$ bisects ∠ BAD (Q.E.D.1)

, ∵ m (∠ CBD) = m (∠ DAC) (2)

(inscribed angles subtended by $\widehat{CD}$)

∴ m (∠ CBH) = m (∠ BAH)

∴ $\overrightarrow{BC}$ is a tangent to the circle passing by the vertices of Δ ABH (Q.E.D.2)

5

[a] ∵ $\overline{AB} // \overline{DC}$, $\overrightarrow{AD}$ is a transversal to them.

∴ m (∠ A) + m (∠ D) = 180° (1)

but ∠ CEH is an exterior angle of the cyclic quadrilateral ABEH

∴ m (∠ CEH) = m (∠ A) (2)

From (1) and (2) :

∴ m (∠ CEH) + m (∠ D) = 180°

∴ HDCE is a cyclic quadrilateral. (Q.E.D.)

[b] ∵ m ($\widehat{BD}$ The major) = 2 m (∠ BCD)

= 2 × 100° = 200°

∴ m ($\widehat{BCD}$) = 360° – 200° = 160°

, ∵ m ($\widehat{HE}$) = m (∠ HME) = 50°

∴ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{BCD}$) – m ($\widehat{HE}$)]

= $\frac{1}{2}$ [160° – 50°] = 55° (The req.)

22 Aswan

1

1 d 2 b 3 a 4 c 5 b 6 c

2

[a] $\because$ $\overleftrightarrow{AB}$ is a tangent to the circle.

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle MAB) = 90°$

In Δ ABM :

$\because (BM)^2 = (AB)^2 + (AM)^2 = (8)^2 + (6)^2 = 100$

$\therefore$ BM = 10 cm.

, $\because$ MA = MD = 6 cm.

$\therefore$ BD = 10 − 6 = 4 cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle BCD) + m(\angle BAD) = 180°$

$\therefore m(\angle BCD) = 180° - 120° = 60°$ (First req.)

, $\because \overrightarrow{BF} \parallel \overline{DC}$, $\overrightarrow{BC}$ is a transversal.

$\therefore m(\angle CBF) = m(\angle BCD) = 60°$ (alternate angles)

$\therefore m(\angle CBE) = 60° + 55° = 115°$

, $\because \angle CBE$ is an exterior angle of a cyclic quadrilateral.

$\therefore m(\angle ADC) = m(\angle CBE) = 115°$ (Second req.)

3

[a] $\because$ D is midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{ME} \perp \overline{AC}$, MD = ME

$\therefore$ AB = AC

$\therefore$ In Δ ABC : $m(\angle ACB) = m(\angle ABC) = 65°$

$\therefore m(\angle BAC) = 180° - (65° + 65°) = 50°$ (The req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 50}{2} = 65°$

, $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle EBC) + m(\angle D) = 180°$

$\therefore m(\angle EBC) = 180° - 115° = 65°$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABE$ (Q.E.D.)

4

[a] $\because$ AB = CD (properties of the rectangle)

, $\because$ CE = CD $\therefore$ AB = CE

$\therefore m(\widehat{AB}) = m(\widehat{CE})$ and adding $m(\widehat{BE})$ to both sides

$\therefore m(\widehat{AE}) = m(\widehat{BC})$

$\therefore$ AE = BC (Q.E.D.)

[b] $\because \overrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency) $= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ AXY (Q.E.D.)

5

[a] $\because m(\angle D) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles substanded by $\widehat{AB}$)

$\therefore m(\angle D) = \frac{1}{2} \times 140° = 70°$ (First req.)

, $\because \overline{AC} \parallel \overline{DB}$, $\overleftrightarrow{AD}$ is transversal

$\therefore m(\angle DAC) + m(\angle D) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle DAC) = 180° - 70° = 110°$ (Second req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore m(\angle BDA) = m(\angle ABD) = 30°$

$\therefore m(\angle A) = 180° - (30° + 30°) = 120°$

, $\because m(\angle DCE) = m(\angle A) = 120°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

23 New valley

1

1 b 2 d 3 d 4 c 5 a 6 b

2

[a] $\because$ ABCD is cyclic quadrilateral.

$\therefore m(\angle ADC) = m(\angle ABE) = 100°$

In Δ ACD :

$\therefore m(\angle ACD) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle CAD) = m(\angle ACD)$

$\therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\quad\therefore m(\angle AXM) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\quad\therefore m(\angle AYM) = 90^\circ$

From the quadrilateral AXMY :

$m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$

(First req.)

, $\because AB = AC$ $\quad\therefore MX = MY$

, $\because MD = MH = r$ $\quad\therefore XD = YH$

(Second req.)

3

[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency)

$= m(\angle ACB)$ (inscribed) (1)

, $\because \overleftrightarrow{XY} \,//\, \overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle DAB) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because m(\angle BCD) = \frac{1}{2}\, m(\angle BMD)$

(inscribed and central angles subteneded by $\widehat{BD}$)

$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$

$\because \overline{AB} \,//\, \overline{CD}$, $\overline{BC}$ is a transversal.

$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$

(alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle ACB) = m(\angle BCD) = 65^\circ$

$\therefore \overrightarrow{CB}$ bisects $\angle ACD$ (First req.)

In ΔABC :

$m(\angle A) = 180^\circ - (65^\circ + 65^\circ) = 50^\circ$ (Second req.)

4

[a] $\because \overline{DE} \,//\, \overline{BC}$

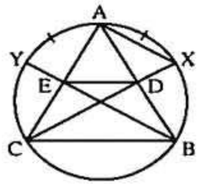
$\therefore m(\widehat{DB}) = m(\widehat{EC})$

adding $m(\widehat{BC})$ to both sides.

$\therefore m(\widehat{DC}) = m(\widehat{EB})$

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

[b] $\because m(\widehat{AX}) = m(\widehat{AY})$



$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on $\overline{ED}$ and on one side of it.

$\therefore$ BCED is a cyclic quadrilateral. (Q.E.D. 1)

, $\because m(\angle DEB) = m(\angle DCB)$

, $\because m(\angle XCB) = m(\angle XAB)$

(two inscribed angles subtended by $\widehat{XB}$)

$\therefore m(\angle DEB) = m(\angle XAB)$ (Q.E.D. 2)

5

[a] State by yourself.

[b] $\because \overline{CD}$ is a diameter in the circle.

$\therefore m(\angle CXD) = 90^\circ$

, $\because \overline{CD} \perp \overline{AB}$

$\therefore m(\angle BEC) = 90^\circ$

$\therefore m(\angle CXD) = m(\angle BEC)$

, $\angle BEC$ is an exterior angle of the figure XYEC

$\therefore$ XYEC is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore m(\angle DYB) = m(\angle XCD)$ (1)

, $\because m(\angle DBX) = m(\angle XCD)$ (2)

(two inscribed angles subtended by $\widehat{XD}$)

From (1) and (2) :

$\therefore m(\angle DYB) = m(\angle DBX)$ (Q.E.D. 2)

24 South Sinai

1

1 a 2 b 3 c 4 d 5 a 6 b

2

[a] $\because m(\widehat{AB}) = 50^\circ$

$\therefore m(\angle D) = \frac{1}{2}\, m(\widehat{AB}) = \frac{1}{2} \times 50^\circ = 25^\circ$

(First req.)

, $m(\angle AMB) = m(\widehat{AB}) = 50^\circ$ (Second req.)

[b] $\because m(\widehat{BC}) = m(\widehat{AD})$

adding $m(\widehat{AC})$ to both sides

$\therefore m(\widehat{AB}) = m(\widehat{CD})$ $\quad\therefore AB = CD$ (Q.E.D.)

3

[a] $\because r_1 = 5$ cm. , $r_2 = 3$ cm.

, $\because r_1 + r_2 = 5 + 3 = 8$ cm.

$\therefore r_1 + r_2 = MN$

$\therefore$ The two circles are touching externally.

[b] $\because \overline{AB}$ is a tangent-segment to the circle.

, $\overline{AC}$ is a diameter of it.

$\therefore \overline{AB} \perp \overline{AC}$

$\therefore m(\angle BAC) = 90°$ (1)

, $\because m(\angle ACD) = \frac{1}{2} m(\angle AMD)$

(inscribed and central angles subtended by $\widehat{AD}$)

$\therefore m(\angle ACD) = \frac{1}{2} \times 60° = 30°$ (2)

In Δ ABC :

$m(\angle ABC) = 180° - (90° + 30°) = 60°$ (First req.)

From (1) and (2) :

$\therefore AB = \frac{1}{2}$ BC (Second req.)

4

[a] In Δ ABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ D is midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

, $\because$ E is midpoint of $\overline{AC}$ $\therefore \overline{ME} \perp \overline{AC}$

$\therefore MD = ME$ (Q.E.D.)

[b] In Δ ABE : $\because AB = AE$

$\therefore m(\angle AEB) = m(\angle B)$

, $\because m(\angle D) = m(\angle B)$

(properties of parallelogram)

$\therefore m(\angle AEB) = m(\angle D)$

$\therefore$ The figure AECD is a cyclic quadrilateral.

(Q.E.D.)

5

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

, $\because$ BCDE is a cyclic quadrilateral.

$\therefore m(\angle EBC) + m(\angle D) = 180°$

$\therefore m(\angle EBC) = 180° - 115° = 65°$

$\therefore m(\angle ABC) = m(\angle EBC)$

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE (Q.E.D. 1)

, $\because m(\angle BEC)$ (inscribed)

$= m(\angle ABC)$ (tangency) $= 65°$

$\therefore m(\angle EBC) = m(\angle BEC)$

$\therefore$ In Δ BCE : CB = CE (Q.E.D. 2)

[b] $\because m(\widehat{BC}) = 2\, m(\angle A) = 2 \times 30° = 60°$

, $\because m(\angle E) = \frac{1}{2}[m(\widehat{AD}) - m(\widehat{BC})]$

$\therefore 50° = \frac{1}{2}[m(\widehat{AD}) - 60°]$

$\therefore 100° = m(\widehat{AD}) - 60°$

$\therefore m(\widehat{AD}) = 160°$ (First req.)

, $\because m(\angle AFD) = \frac{1}{2}[m(\widehat{AD}) + m(\widehat{BC})]$

$\therefore m(\angle AFD) = \frac{1}{2}[160° + 60°] = 110°$

(Second req.)

25 North Sinai

1

1 c 2 a 3 b 4 b 5 c 6 c

2

[a] $\because AB = CD$, $\overline{MW} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

$\therefore MX = MY$

, $\because MW = MH = r$

$\therefore WX = HY$ (Q.E.D.)

[b] $\because \overrightarrow{CD} // \overline{BA}$ $\therefore m(\widehat{AC}) = m(\widehat{BC})$

$\therefore AC = BC$ (First req.)

, $\because \overline{AB}$ is a diameter of the circle

$\therefore m(\angle ACB) = 90°$

In ΔABC : $\therefore m(\angle B) = m(\angle A) = \frac{180° - 90°}{2} = 45°$

(Second req.)

3

[a] State by yourself.

[b] $\because$ D is the midpoint of $\overline{BW}$

$\therefore \overline{MD} \perp \overline{BW}$

$\therefore m(\angle WDM) = 90°$

, $\because \overrightarrow{AC}$ is a tangent to the circle

$\therefore \overline{AC} \perp \overline{BC}$ $\therefore m(\angle ACM) = 90°$

$\therefore m(\angle WDM) + m(\angle ACM) = 180°$

$\therefore$ The figure ADMC is a cyclic quadrilateral. (Q.E.D. 1)

, $\because \angle$ CMH is an exterior angle of the cyclic quadrilateral ADMC

$\therefore m(\angle CMH) = m(\angle A)$ (1)

, $\because m(\angle CBH) = \frac{1}{2} m(\angle CMH)$ (2)

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

From (1) and (2) :

$\therefore m(\angle CBH) = \frac{1}{2} m(\angle A)$ (Q.E.D. 2)

4

[a] $\because m(\angle A) = \frac{1}{2}[m(\overset{\frown}{CH}) - m(\overset{\frown}{BD})]$

$\therefore 30^\circ = \frac{1}{2}[80^\circ - m(\overset{\frown}{BD})]$

$\therefore 60^\circ = 80^\circ - m(\overset{\frown}{BD})$

$\therefore m(\overset{\frown}{BD}) = 80^\circ - 60^\circ = 20^\circ$

, $\because \overline{BC}$ is a diameter in the circle

$\therefore m(\overset{\frown}{BC}) = 180^\circ$

$\therefore m(\overset{\frown}{DH}) = 360^\circ - [180^\circ + 20^\circ + 80^\circ] = 80^\circ$ (The req.)

[b] $\because m(\angle BDC)$ (inscribed) $= m(\angle ABC)$ (tangency) $= 70^\circ$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = 70^\circ$

In ΔABC :

$\therefore m(\angle BAC) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$ (The req.)

5

[a] In ΔABD : $\because AB = AD$

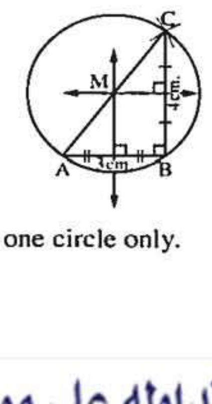
$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$

$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b]



We can draw one circle only.

26 Red Sea

1

1 c 2 b 3 a 4 d 5 c 6 c

2

[a] $\because AB = CD$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore MX = MY$

, $\because MH = MF = r$ $\therefore HX = FY$ (Q.E.D.)

[b] $\because m(\angle ADB) = \frac{1}{2} m(\overset{\frown}{AB}) = \frac{1}{2} \times 110^\circ = 55^\circ$

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle HBC) = m(\angle CDB) + m(\angle ADB)$
$= 30^\circ + 55^\circ = 85^\circ$ (The req.)

3

[a] In ΔBMC : $\because MB = MC = r$

$\therefore m(\angle MCB) = m(\angle MBC) = 25^\circ$

$\therefore m(\angle BMC) = 180^\circ - (25^\circ + 25^\circ) = 130^\circ$

, $\because m(\angle BAC) = \frac{1}{2} m(\angle BMC)$

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

$\therefore m(\angle BAC) = \frac{1}{2} \times 130^\circ = 65^\circ$ (The req.)

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = 50^\circ$

$\therefore m(\angle A) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

, $\because m(\angle A) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$

$\therefore$ ABDC is a cyclic quadrilateral . (Q.E.D.)

4

[a] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

$\therefore \overline{AB} \perp \overline{MN}$ $\therefore m(\angle AXN) = 90^\circ$

$\because$ The sum of the measures of the interior angles of the quadrilateral CDNX = 360°

$\therefore m(\angle CDN) = 360^\circ - (125^\circ + 55^\circ + 90^\circ) = 90^\circ$

$\therefore \overline{ND} \perp \overline{CD}$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because \overleftrightarrow{AX}$ is a common tangent for two circles

$\therefore m(\angle BDA)$ (inscribed)
$= m(\angle BAX)$ (tangency)

, m (∠ CHA) (inscribed)
= m (∠ CAX) (tangency)

∴ m (∠ BDA) = m (∠ CHA)

and they are corresponding angles

∴ $\overline{BD}$ // $\overline{CH}$ (Q.E.D.)

5

[a] ∵ m ($\widehat{BD}$) = 2 m (∠ C)

∴ m ($\widehat{BD}$) = 2 × 26° = 52°

∵ m (∠ A) = $\frac{1}{2}$ [m ($\widehat{CH}$) – m ($\widehat{BD}$)]

∴ 40° = $\frac{1}{2}$ [m ($\widehat{CH}$) – 52°]

∴ 80° = m ($\widehat{CH}$) – 52°

∴ m ($\widehat{CH}$) = 80° + 52° = 132° (First req.)

∴ m (∠ HXC) = $\frac{1}{2}$ [m ($\widehat{CH}$) + m ($\widehat{BD}$)]

= $\frac{1}{2}$ [132° + 52°] = 92° (Second req.)

[b] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ m (∠ ABC) = m (∠ ACB) = $\frac{180° - 70°}{2}$ = 55°

∴ m (∠ BHC) (inscribed)
= m (∠ ABC) (tangency) = 55°

, ∵ BCDH is a cyclic quadrilateral.

∴ m (∠ CBH) + m (∠ CDH) = 180°

∴ m (∠ CBH) = 180° – 125° = 55°

In Δ BCH : ∴ m (∠ BHC) = m (∠ CBH)

∴ CB = CH (Q.E.D.)

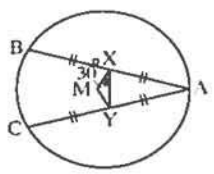
27 Matrouh

1

1 c	2 c	3 d	4 b	5 c	6 b

2

[a]



∵ X is the midpoint of $\overline{AB}$

∴ $\overline{MX}$ ⊥ $\overline{AB}$

, ∵ Y is the midpoint of $\overline{AC}$

∴ $\overline{MY}$ ⊥ $\overline{AC}$

, ∵ AB = AC ∴ MX = MY

∴ Δ MXY is an isoscles triangle. (Q.E.D.)

[b] ∵ $\overrightarrow{AF}$ // $\overline{DE}$, $\overleftrightarrow{AB}$ is a transversal.

∴ m (∠ BAF) = m (∠ AED)
(alternate angles) (1)

, ∵ m (∠ C) (inscribed)
= m (∠ BAF) (tangency) (2)

From (1) and (2) : ∴ m (∠ C) = m (∠ AED)

∴ DEBC is a cyclic quadrilateral. (Q.E.D.)

3

[a] ∵ m (∠ D) = $\frac{1}{2}$ m (∠ M)

(inscribed and central angles subtended by $\widehat{BC}$)

∴ m (∠ D) = $\frac{1}{2}$ × 100° = 50°

, ∵ ∠ ABD is an exterior angle of Δ BCD

∴ m (∠ ABD) = m (∠ BDC) + m (∠ DCB)

∴ m (∠ DCB) = 120° – 50° = 70° (The req.)

[b] ∵ $\overleftrightarrow{CA}$ and $\overleftrightarrow{CB}$ are two tangents to the circle.

∴ $\overline{MA}$ ⊥ $\overline{AC}$ ∴ m (∠ MAC) = 90°

, $\overline{MB}$ ⊥ $\overline{BC}$

∴ m (∠ MBC) = 90°

∴ m (∠ MAC) + m (∠ MBC) = 180°

∴ ACBM is a cyclic quadrilateral.

, ∵ ∠ DMB is an exterior angle of it

∴ m (∠ DMB) = m (∠ ACB) (Q.E.D.)

4

[a] ∵ $\overleftrightarrow{AD}$ is a tangent to the circle.

∴ m (∠ DAB) (tangency)
= m (∠ ACB) (inscribed) (1)

, ∵ $\overleftrightarrow{XY}$ // $\overline{BC}$, $\overleftrightarrow{YC}$ is a transversal.

∴ m (∠ AYX) = m (∠ ACB)
(corresponding angles) (2)

From (1) and (2) : ∴ m (∠ DAB) = m (∠ AYX)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$

∴ m ($\widehat{DB}$) = m ($\widehat{EC}$) adding m ($\widehat{BC}$) to both sides

∴ m ($\widehat{DC}$) = m ($\widehat{EB}$)

∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

5

[a] Prove by yourself.

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 70°}{2} = 55°$

$\therefore m(\angle CEB)$ (inscribed)

$= m(\angle CBA)$ (tangency) $= 55°$

, $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) + m(\angle CDE) = 180°$

$\therefore m(\angle CBE) = 180° - 125° = 55°$

In Δ EBC : $\therefore m(\angle CEB) = m(\angle CBE)$

$\therefore CB = CE$ (Q.E.D.1)

, $\because m(\angle ACB) = m(\angle CBE) = 55°$

and they are alternate angles

$\therefore \overrightarrow{AC} \parallel \overline{BE}$ (Q.E.D.2)

Governorates' Examinations

1 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

(1) The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc.

(a) half (b) third (c) quarter (d) double

(2) It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) parallelogram. (c) rectangle. (d) rhombus.

(3) The centre of the inscribed circle of any triangle is the point of intersection of its ....

(a) altitudes. (b) medians.

(c) axes of symmetry of its sides. (d) bisectors of its interior angles.

(4) If the two circles M and N are touching internally , the radius length of one of them = 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm.

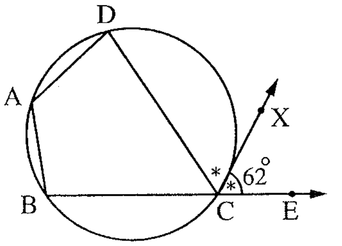
(a) 12 (b) 11 (c) 6 (d) 5

(5) **In the opposite figure :**

If $E \in \overrightarrow{BC}$, $\overrightarrow{CX}$ bisects $\angle$ DCE

, m ($\angle$ XCE) = 62°

, then m ($\angle$ A) = ...

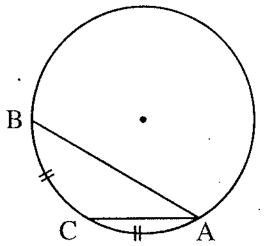


(a) 62° (b) 118° (c) 56° (d) 124°

(6) **In the opposite figure :**

If C is the midpoint of $\overset{\frown}{AB}$

, then AB .... 2 AC



(a) < (b) > (c) ≥ (d) =

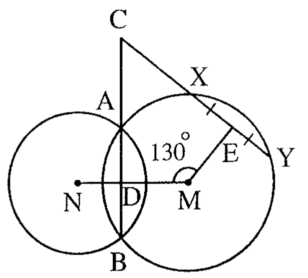
---

2 [a] In the opposite figure :

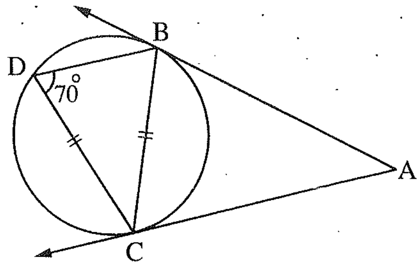
If E is the midpoint of $\overline{XY}$

, m ($\angle$ EMN) = 130°

, then find : m ($\angle$ C)



[b] In the opposite figure :



If $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

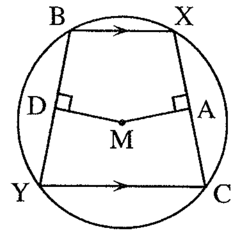
, m (∠ D) = 70° , CB = CD

(1) **Find :** m (∠ A)

(2) **Prove that :** $\overline{BD}$ // $\overline{AC}$

---

3 **[a] In the opposite figure :**

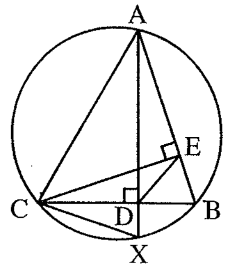


$\overline{XB}$ // $\overline{CY}$, $\overline{MA} \perp \overline{XC}$

, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD

[b] In the opposite figure :



$\overline{CE} \perp \overline{AB}$, $\overrightarrow{AD} \perp \overline{BC}$ and intersects the circle at X

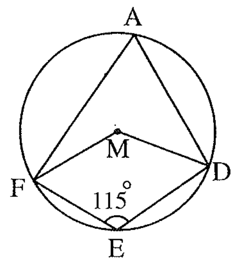
Prove that :

(1) AEDC is a cyclic quadrilateral.

(2) $\overrightarrow{CB}$ bisects ∠ ECX

---

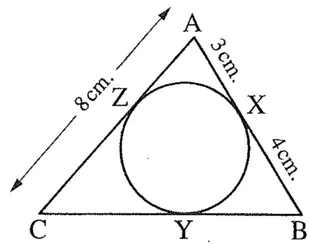
4 **[a] In the opposite figure :**



If m (∠ DEF) = 115°

, **then find :** m (∠ DMF)

[b] In the opposite figure :



Inscribed circle of the triangle ABC touches

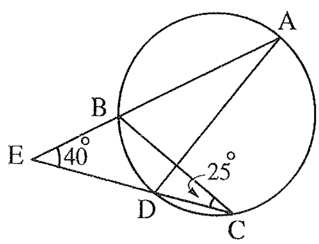
its sides at X , Y and Z

If AX = 3 cm. , XB = 4 cm. , AC = 8 cm.

Find : The length of $\overline{BC}$

---

5 **[a] In the opposite figure :**

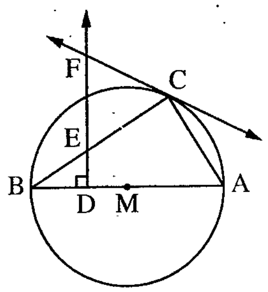


$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ C) = 25°

, m (∠ E) = 40°

Find : m (∠ ADC)

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{CF}$ is a tangent to the circle at C

, $\overrightarrow{DF} \perp \overline{AB}$ and intersects $\overline{BC}$ at E

Prove that :

(1) ADEC is a cyclic quadrilateral.

(2) Δ FCE is an isosceles triangle.

2 Alexandria Governorate

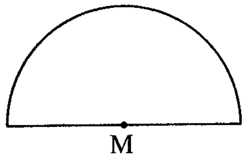
Answer the following questions :

1 Choose the correct answer from those given :

(1) The two opposite angles in the cyclic quadrilateral are ....

(a) equal. (b) supplementary. (c) complementary. (d) alternate.

(2) The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.



(a) $2 \pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$

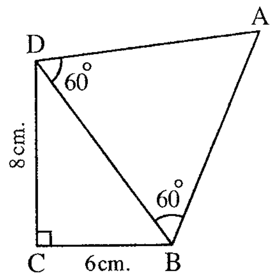
(3) In a regular hexagon , the measure of the angle of its vertex equals ....

(a) $60°$ (b) $108°$ (c) $120°$ (d) $135°$

(4) If $\overline{AB}$ is a line segment , then the number of circles can be drawn passing through A and B equals ....

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(5) **In the opposite figure :**



The length of $\overline{AB}$ = ... cm.

(a) $10\sqrt{3}$ (b) 10

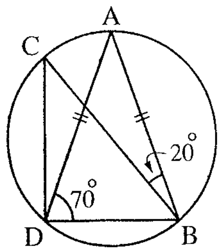
(c) 5 (d) $5\sqrt{3}$

(6) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

---

2 [a] In the opposite figure :



AB = AD

, m ($\angle$ ABC) = $20°$

, m ($\angle$ ADB) = $70°$

Find : m ($\angle$ C) , m ($\angle$ BDC)

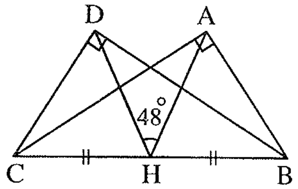
[b] In the opposite figure :

$m(\angle BAC) = m(\angle BDC) = 90°$

, H is the midpoint of $\overline{BC}$ and $m(\angle AHD) = 48°$

(1) **Prove that :** ABCD is a cyclic quadrilateral.

(2) **Find :** $m(\angle ABD)$



---

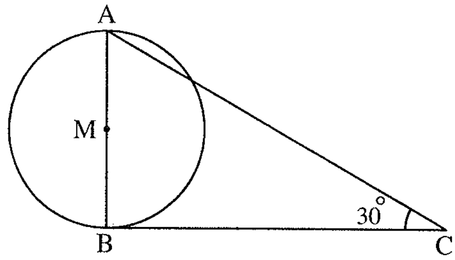
3 **[a] In the opposite figure :**

A circle M of circumference 44 cm.

, $\overline{AB}$ is a diameter , $\overline{BC}$ is a tangent at B

and $m(\angle ACB) = 30°$

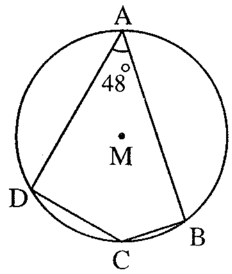
Find : The length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$



[b] In the opposite figure :

If M is a circle , $m(\angle A) = 48°$

Find : $m(\overset{\frown}{BD}$ the major)



---

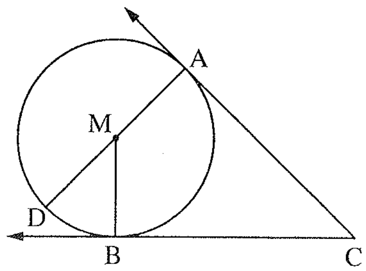
4 **[a] In the opposite figure :**

$\overline{AD}$ is a diameter in a circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M ,

touch it at A and B respectively.

Prove that : $m(\angle DMB) = m(\angle ACB)$



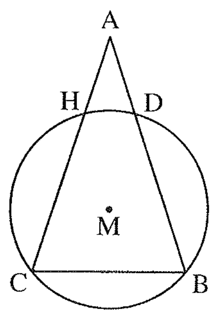
[b] In the opposite figure :

ABC is a triangle in which AB = AC

, $\overline{BC}$ is a chord in the circle M

, if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that : $m(\overset{\frown}{DB}) = m(\overset{\frown}{HC})$



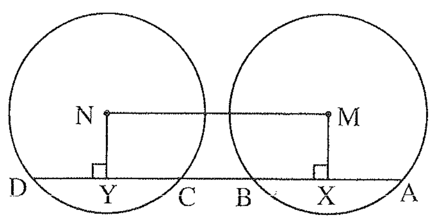
---

5 **[a] In the opposite figure :**

M and N are two congruent circles

, AB = CD

Prove that : The figure MXYN is a rectangle.



[b] ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and $\overrightarrow{HA}$ and $\overrightarrow{HB}$ are two tangents to the circle at A and B , if m (∠ AHB) = 70° and m (∠ ADC) = 125° , **prove that :**

(1) AB = AC

(2) $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and H

3 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) If the area of the circle is 9 π cm^2 , then its radius length = ... cm.

(a) 9 (b) 2 (c) (– 3) (d) 3

(2) The number of symmetric axes of a square = ...

(a) 1 (b) 2 (c) 3 (d) 4

(3) If M is a circle of a diameter length equals 14 cm. , MA = (2 X + 3) cm. where A lies on the circle , then X = ...

(a) 5 (b) 3 (c) 2 (d) 1

(4) The raito between the measure of the inscribed angle and the measure of the central angle subtended by the same arc = ...

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

(5) If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{2}$ m (∠ D) , then m (∠ B) = ...

(a) 90° (b) 60° (c) 120° (d) 180°

(6) If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

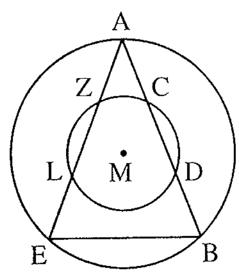
---

2 [a] In the opposite figure :

Two concentric circles at M

, m (∠ ABE) = m (∠ AEB)

Prove that : CD = ZL

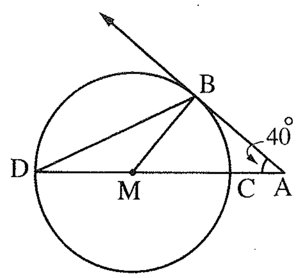


[b] In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M

, m (∠ A) = 40°

Find with proof : m (∠ BDC)



---

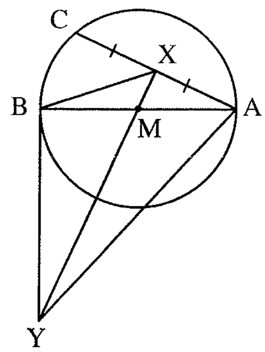
3 **[a]** Using your geometric tools , draw $\overline{AB}$ with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.
What are the possible solutions ? (Don't remove the arcs)

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, X is the midpoint of $\overline{AC}$ and $\overrightarrow{XM}$ intersecting the tangnet of the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.



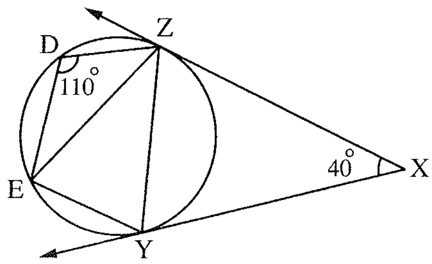
---

4 **[a] In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at the two points Y and Z , m (∠ X) = 40°

, m (∠ D) = 110°

Prove that : m (∠ ZYE) = m (∠ ZEY)

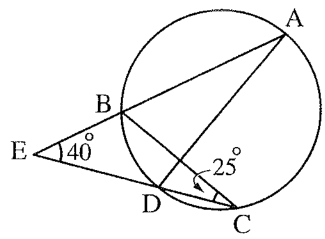


[b] In the opposite figure :

m (∠ E) = 40° , m (∠ C) = 25°

Find with proof :

(1) m (∠ ADC) (2) m ($\overset{\frown}{AC}$)



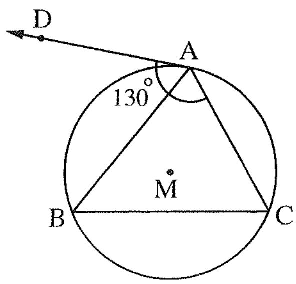
---

5 **[a] In the opposite figure :**

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof : m (∠ B)



[b] ABCD is a quadrilateral drawn in a circle , E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$

, m ($\overset{\frown}{AB}$) = 110° , m (∠ CBE) = 85°

Find with proof : m (∠ BDC)

4 El-Sharkia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

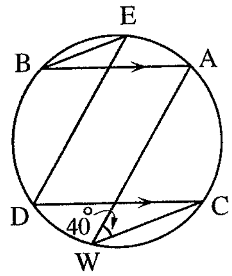
(1) The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) perpendicular. (c) coincide. (d) intersecting.

(2) The number of the axes of symmetry of the semicircle .... the number of the axes of symmetry of the isosceles triangle.

(a) > (b) < (c) = (d) ≥

(3) **In the opposite figure :**



$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40°

(c) 30° (d) 45°

(4) A circle , its radius length $(2X + 6)$ cm. and the straight line L is at distance $(X + 2)$ cm. from its centre where $X > 0$, then L is ....

(a) outside the circle. (b) a tangent to the circle.

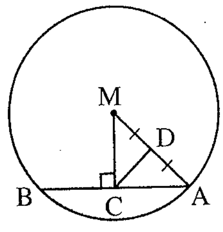
(c) a secant to the circle. (d) passing through the centre.

(5) If the straight line $\overleftrightarrow{AB} \cap$ the circle M = {A , B}

, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) {A , B} (b) $\overline{AB}$ (c) $\overrightarrow{AB}$ (d) $\overrightarrow{BA}$

(6) **In the opposite figure :**



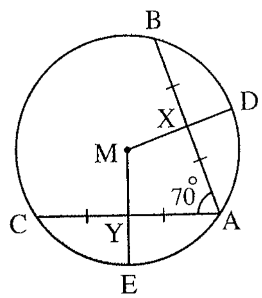
CD = 3 cm. , $\overline{MC} \perp \overline{AB}$

, D is the midpoint of $\overline{MA}$

then the area of the circle M = ... π cm².

(a) 3 (b) 6 (c) 9 (d) 36

2 [a] **In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

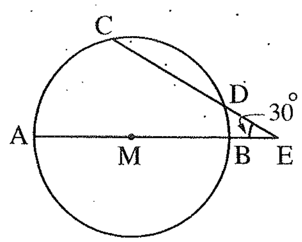
(1) **Find :** m (∠ DME) (2) **Prove that :** XD = YE

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ E) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$



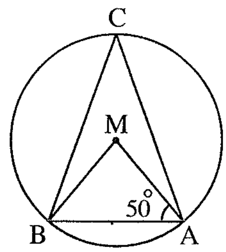
---

3 **[a] Complete :** The measure of the inscribed angle equals .... the measure of the central angle .... by the same arc.

[b] In the opposite figure :

M is a circle , m (∠ MAB) = 50°

Find : m (∠ C)



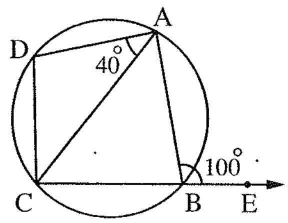
---

4 **[a] In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : Δ DAC is an isosceles triangle.

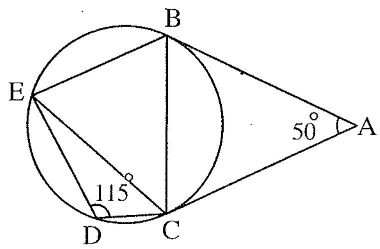


[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ A) = 50° , m (∠ D) = 115°

Prove that : (1) $\overrightarrow{BC}$ bisects ∠ ABE (2) CB = CE



---

5 **[a] Complete :** The measure of the inscribed angle in a semicircle equals ....°

[b] In the opposite figure :

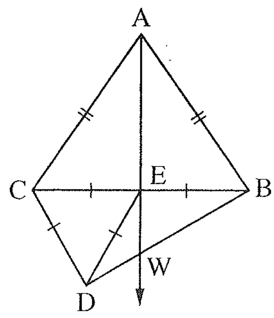
ABC and DCE are two equilateral triangles

, E is the midpoint of $\overline{BC}$, $\overrightarrow{AE} \cap \overline{BD} = \{W\}$

(1) **Prove that :** $\overline{AC}$ is a tangent-segment to the circle which passes through the vertices of Δ CED

(2) **Prove that :** CDWE is a cyclic quadrilateral.

(3) **Find :** The centre of the circle which passes through the vertices of the quadrilateral CDWE

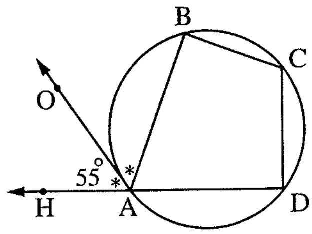


5 El-Monofia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer :

(1) **In the opposite figure :**



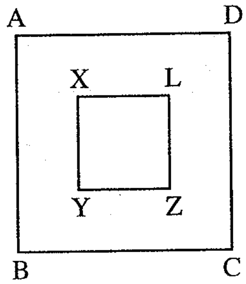
$H \in \overrightarrow{DA}$, $\overrightarrow{AO}$ bisects $\angle$ HAB

, m ($\angle$ HAO) = 55°

, then m ($\angle$ C) = ...

(a) 55° (b) 75° (c) 110° (d) 125°

(2) **In the opposite figure :**



If the side length of the square ABCD = 7 cm.

and the side length of the square XYZL = 3 cm.

, then the area of the shaded part = ... cm^2.

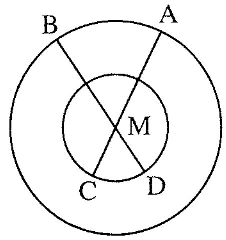
(a) (7 – 3) (b) 4 (7 – 3)

(c) $(7-3)^2$ (d) $(7^2 - 3^2)$

(3) If $\overleftrightarrow{AB} \cap$ the circle M = {A , B} , then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) {A , B} (d) $\overrightarrow{AB}$

(4) **In the opposite figure :**



Two concentric circles with centre M

, the radii lengths of them are 6 cm. and 3 cm.

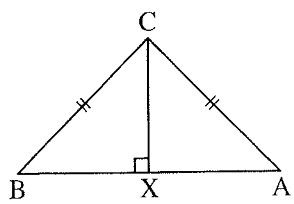
, if m $(\overset{\frown}{AB})$ = 60° , then m $(\overset{\frown}{DC})$ = ...

(a) 60° (b) 30° (c) 120° (d) 40°

(5) If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in a circle M and the area of triangle AMB = 8 cm^2. , then the length of radius of this circle = ...

(a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.

(6) **In the opposite figure :**



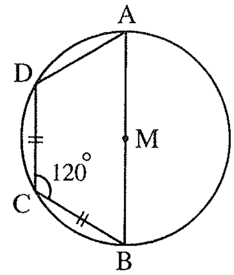
CA = CB , $\overline{CX} \perp \overline{AB}$

, AB = 2 CX

, then m ($\angle$ A) = ...

(a) 30° (b) 60° (c) 90° (d) 45°

2 **[a] In the opposite figure :**



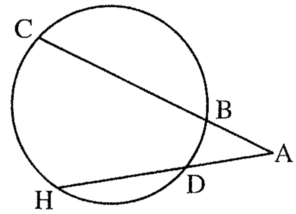
ABCD is a quadrilateral inscribed in the circle M

, M ∈ $\overline{AB}$, CB = CD

, m (∠ BCD) = 120°

Find : (1) m (∠ A) (2) m (∠ D)

[b] In the opposite figure :

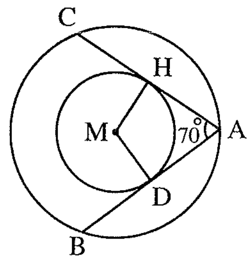


If m ($\overset{\frown}{HC}$) = 100° , m ($\overset{\frown}{BD}$) = 30°

Find : m (∠ A)

---

3 **[a] In the opposite figure :**



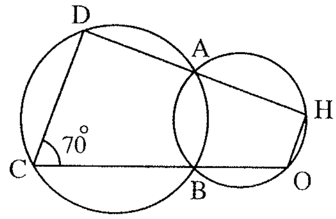
Two concentric circles at M

, $\overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle

, m (∠ A) = 70°

(1) **Find :** m (∠ DMH) (2) **Prove that :** AB = AC

[b] In the opposite figure :



Two intersecting circles at A and B , m (∠ C) = 70°

(1) **Find :** m (∠ O)

(2) **Prove that :** $\overline{CD}$ // $\overline{HO}$

---

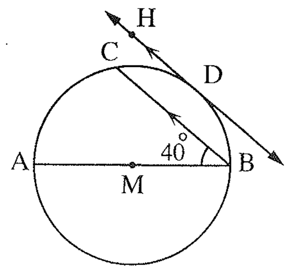
4 [a] $\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord such that m (∠ BAC) = 30°

, draw $\overline{BC}$ and draw $\overline{MD}$ ⊥ $\overline{AC}$ and cut it at D

(1) **Prove that :** $\overline{MD}$ // $\overline{BC}$

(2) **Porve that :** The length $\overline{BC}$ = the length of the radius of this circle.

[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, m (∠ B) = 40° , $\overleftrightarrow{DH}$ is a tangent at D

, $\overleftrightarrow{DH}$ // $\overline{BC}$

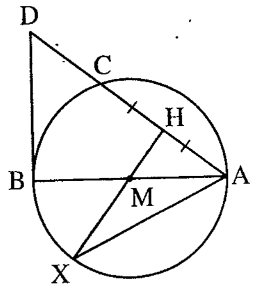
Find : m ($\overset{\frown}{DC}$)

---

5 [a] If circle with radius length 5 cm. , A is a point in its plane where MA = (2 x – 3) cm.

Find the value of x if A is located outside the circle.

[b] In the opposite figure :



$\overline{AB}$ is a diameter of the circle M , H is a midpoint of a chord $\overline{AC}$

, $\overline{BD}$ is a tangent to the circle at B

, $\overrightarrow{HM}$ cuts the circle at X , **porve that :**

(1) MHDB is a cyclic quadrilateral.

(2) m ($\angle$ BAX) = $\frac{1}{2}$ m ($\angle$ D)

(3) $\overleftrightarrow{AB}$ is a tangent to the circle passing through the points B , C and D

6 El-Gharbia Governorate

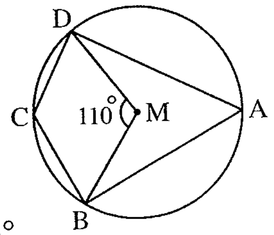
Answer the following questions :

1 Choose the correct answer from those given :

(1) If the length of a diameter of a circle is 8 cm. and the straight line L at a distance of 4 cm. from its centre , then L is ....

(a) a secant to the circle at two points.
(b) lying outside the circle.
(c) a tangent to the circle.
(d) an axis of symmetry to the circle.

(2) **In the opposite figure :**



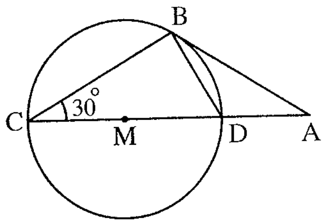
If M is the centre of the circle

, m ($\angle$ BMD) = 110°

, then m ($\angle$ C) = ...

(a) 70°
(b) 110°
(c) 125°
(d) 55°

(3) **In the opposite figure :**



$\overline{AB}$ is a tangent of the circle M

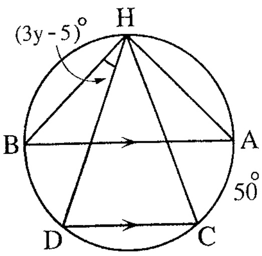
, then m ($\angle$ ABC) = ...

(a) 120°
(b) 110°
(c) 90°
(d) 30°

(4) The centre of the inscribed circle of any triangle is the intersection point ....

(a) its medians.
(b) its heights.
(c) the symmetric axes of its sides.
(d) bisectors of its interior angles.

(5) **In the opposite figure :**



m ($\overset{\frown}{AC}$) = 50° , $\overline{AB}$ // $\overline{CD}$

, then the value of y = ...

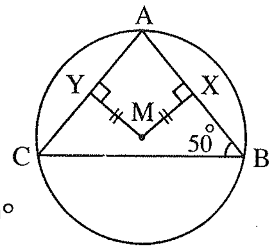
(a) 5°
(b) 10°
(c) 15°
(d) 20°

(6) **In the opposite figure :**

MX = MY , m (∠ B) = 50°

, then m (∠ A) = ...

(a) 50° (b) 60° (c) 70° (d) 80°



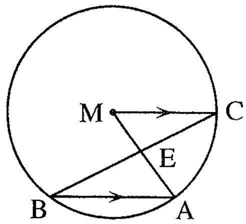
---

2 **[a] In the opposite figure :**

$\overline{AB}$ is a chord in the circle M

, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE



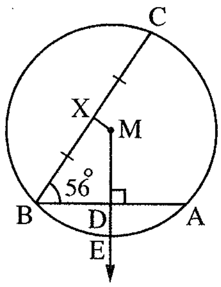
[b] In the opposite figure :

$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M

, its radius of length 5 cm. , $\overrightarrow{MD} \perp \overline{AB}$ and cuts $\overline{AB}$ at D and cuts the circle at E , X is midpoint of $\overline{BC}$

, AB = 8 cm. and m (∠ ABC) = 56°

Find : (1) m (∠ DMX) (2) The length of $\overline{DE}$



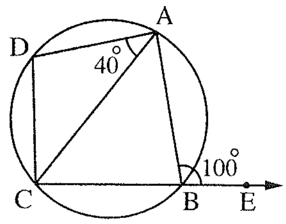
---

3 **[a] In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that : m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$



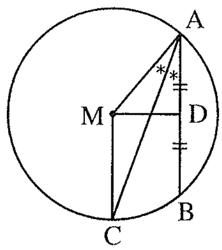
[b] In the opposite figure :

$\overline{AB}$ is a chord in the circle M

, $\overrightarrow{AC}$ bisects ∠ BAM and cuts the circle M at C

, D is midpoint of $\overline{AB}$

Prove that : $\overline{DM} \perp \overline{CM}$



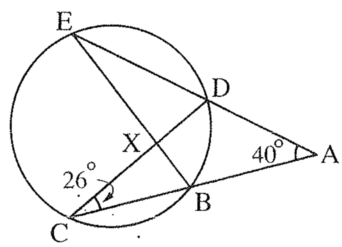
---

4 **[a] In the opposite figure :**

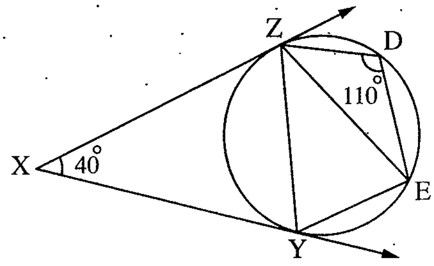
$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m (∠ A) = 40°

, $\overline{DC} \cap \overline{BE} = \{X\}$, m (∠ BCD) = 26°

Find : (1) m $(\overset{\frown}{CE})$ (2) m (∠ EXC)



[b] In the opposite figure :



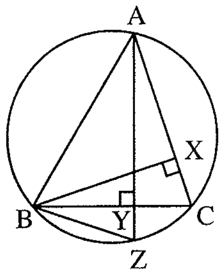
$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X , m ($\angle$ X) = 40°

, m ($\angle$ D) = 110°

Prove that : m $(\overset{\frown}{ZDE})$ = m $(\overset{\frown}{ZY})$

---

5 **[a] In the opposite figure :**



ABC is a triangle drawn in a circle

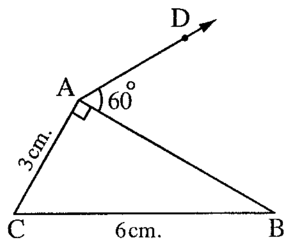
, $\overline{BX} \perp \overline{AC}$, $\overrightarrow{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

(1) ABYX is a cyclic quadrilateral.

(2) $\overrightarrow{BC}$ bisects $\angle$ XBZ

[b] In the opposite figure :



ABC is a right-angled triangle at A

, AC = 3 cm. , BC = 6 cm.

, m ($\angle$ BAD) = 60°

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC

7 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **[a] Choose the correct answer from the given answers :**

(1) M and N are two circles of radii lengths 9 cm. , 4 cm. , MN = 5 cm.

, then the two circles are ....

(a) intersecting. (b) touching internally.

(c) touching externally. (d) distant.

(2) The centres of all circles passing through the points A and B lie on ....

(a) $\overline{AB}$ (b) midpoint of $\overline{AB}$

(c) the symmetry axis of $\overline{AB}$ (d) the perpendicular to $\overline{AB}$ from B

(3) The measure of the inscribed angle which is drawn in a semicircle equals ....

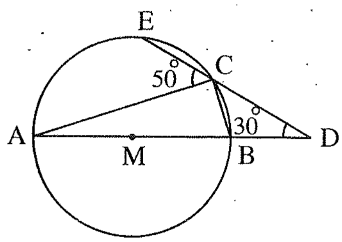
(a) 180° (b) 90° (c) 45° (d) 100°

[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m (∠ D) = 30° , m (∠ ACE) = 50°

Find by proof : m (∠ CBA)



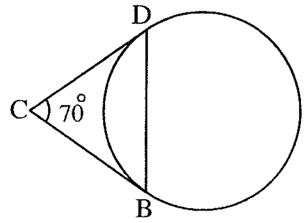
2 **[a] Choose the correct answer from the given answers :**

(1) **In the opposite figure :**

$\overline{CB}$ and $\overline{CD}$ are two tangent-segments at B and D

, m (∠ C) = 70°

, then m ($\overset{\frown}{DB}$ the minor) = ...



(a) 180° (b) 90° (c) 100° (d) 110°

(2) $\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle M , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively , MX = 3 cm. , then MY = ... cm.

(a) 3 (b) 6 (c) $\frac{3}{2}$ (d) 4

(3) The length of the arc which represents $\frac{1}{4}$ of the circle equals ....

(a) $4\pi r$ (b) $2\pi r$ (c) πr (d) $\frac{1}{2}\pi r$

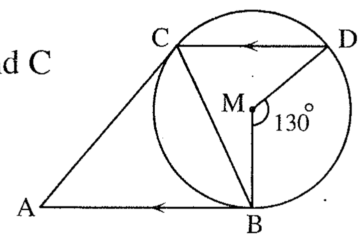
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

(2) **Find by proof :** m (∠ A)



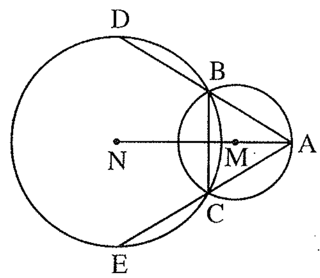
3 **[a]** Using the geometric tools , draw $\overline{AB}$ with length 6 cm. , then draw $\overrightarrow{AC}$ where m (∠ CAB) = 60° , draw the circle that passes through the points A , B and its centre lies on $\overrightarrow{AC}$ and calculate the length of its radius (Don't reomve the arcs).

[b] In the opposite figure :

M and N are two intersecting circles at B and C

, A ∈ $\overleftrightarrow{MN}$

Prove that : BD = CE



4 **[a] In the opposite figure :**

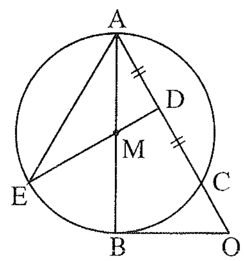
$\overline{OB}$ is a tangent-segment to the circle M at B

, $\overline{AB}$ is a diameter , D is the midpoint of $\overline{AC}$

Prove that :

(1) DOBM is a cyclic quadrilateral.

(2) m (∠ AOB) = 2 m (∠ BAE)



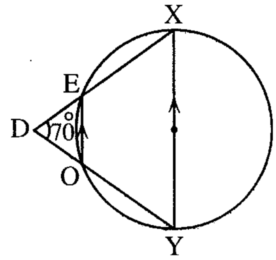
[b] In the opposite figure :

$\overline{XY}$ is a diameter in the circle

, $\overline{EO}$ is a chord in it , where $\overline{XY} \parallel \overline{EO}$

, m (∠ D) = 70°

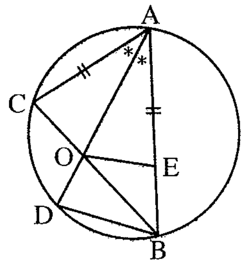
Find : m $(\overset{\frown}{EX})$



5 **[a] In the opposite figure :**

AE = AC , $\overrightarrow{AD}$ bisects ∠ BAC

Prove that : EBDO is a cyclic quadrilateral.



[b] $\overline{AB}$ is a diameter in a circle , $\overline{AC}$ is a chord in it , m (∠ CAB) = 30°

, draw $\overrightarrow{AC}$ to cut the tangent to the circle at B at D.

Prove that : $\overrightarrow{BA}$ touches the circle passing through the vertices of the triangle BCD

8 Ismailia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

(1) A circle its radius length is 5 cm. , then its circumference = ... cm.

(a) 5 π (b) 7 π (c) 10 π (d) 25 π

(2) We can draw a circle passes through the vertices of ....

(a) rectangle. (b) rhombus. (c) trapezium. (d) parallelogram.

(3) The number of axes of symmetry of the circle = ...

(a) one axis. (b) two axes.

(c) three axes. (d) an infinite number of axes.

(4) M is a circle with radius length r , $\overleftrightarrow{MA} \perp$ straight line L where $\overleftrightarrow{MA} \cap L = \{A\}$

If MA > r , then L is ....

(a) a tangent to the circle. (b) a diameter in the circle.

(c) outside the circle. (d) a secant to the circle.

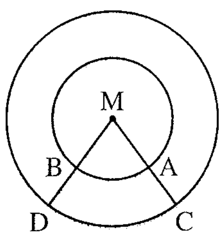
(5) **In the opposite figure :**

Two concentric circles.

If the lengths of their radii are 2 cm. and 5 cm.

, then $\frac{m(\overset{\frown}{AB})}{m(\overset{\frown}{CD})}$ = ...

(a) $\frac{2}{5}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

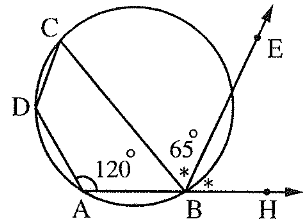


(6) The sum of measures of the interior angles of the quadrilateral = ...

(a) 90° (b) 180° (c) 270° (d) 360°

---

2 **[a] In the opposite figure :**



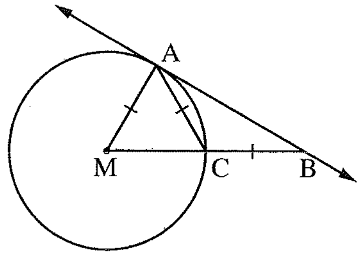
ABCD is a cyclic quadrilateral in which

m (∠ A) = 120° , $\overrightarrow{BE}$ bisects ∠ HBC

, m (∠ EBC) = 65°

Find with proof : (1) m (∠ C) (2) m (∠ D)

[b] In the opposite figure :



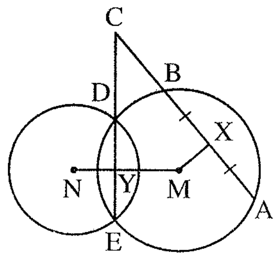
M is a circle , AM = AC = BC

Prove that :

$\overleftrightarrow{AB}$ is a tangent to the circle at A

---

3 **[a] In opposite figure :**

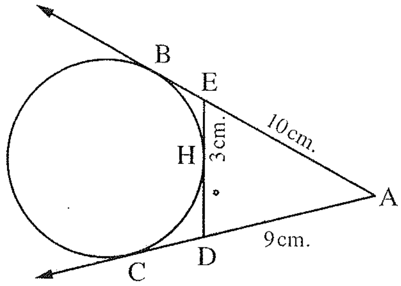


X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$

(1) **Prove that :** CXMY is a cyclic quadrilateral.

(2) **Find :** The centre of the circle which passes through the vertices of the figure CXMY

[b] In the opposite figure :



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to a circle

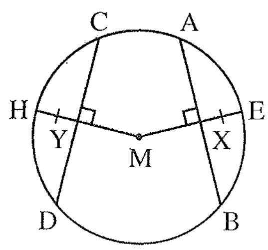
, $\overline{ED}$ is a tangent to the circle at H such that AE = 10 cm.

, EH = 3 cm. , AD = 9 cm.

Find : The length of $\overline{ED}$

---

4 **[a] In the opposite figure :**



$\overline{ME} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

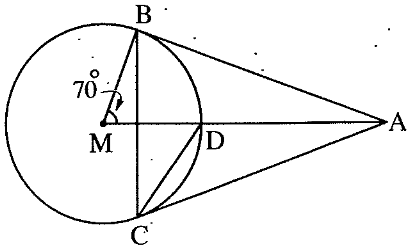
, EX = YH

Prove that : AB = CD

[b] Using geometric tools. Draw $\overline{AB}$ its length is 6 cm. , then draw a circle passing through the two points A , B and its radius length is 3 cm.

How many circles can be drawn ?

5 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A

, m (∠ AMB) = 70°

Find : (1) m (∠ ABC) (2) m (∠ ACD)

[b] $\overline{AB}$ and $\overline{CD}$ are two equal chords in length in a circle

, $\overline{AB} \cap \overline{CD} = \{E\}$, m $(\widehat{AD})$ = 50°

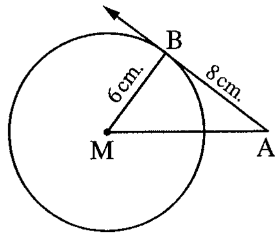
(1) **Prove that :** m $(\widehat{AD})$ = m $(\widehat{BC})$ (2) **Find :** m (∠ AED)

9 Suez Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

(1) **In the opposite figure :**



$\overrightarrow{AB}$ is a tangent to the circle M

, MB = 6 cm. , AB = 8 cm.

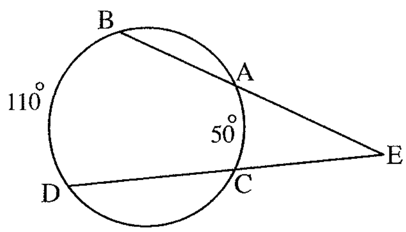
, then AM = ... cm.

(a) 5 (b) 10 (c) 12 (d) 13

(2) If the two circles M and N are touching externally, the radius length of one of them is 5 cm. , and MN = 9 cm. , then the radius length of the other circle equals .... cm.

(a) 4 (b) 5 (c) 9 (d) 14

(3) **In the opposite figure :**



If m $(\widehat{AC})$ = 50° , m $(\widehat{BD})$ = 110°

, then m (∠ E) = ...°

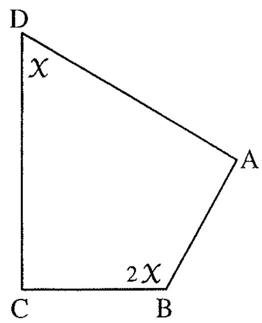
(a) 60 (b) 50

(c) 40 (d) 30

(4) A circle can be drawn passing the vertices of a ....

(a) rhombus. (b) rectangle. (c) trapezoid. (d) parallelogram.

(5) **In the opposite figure :**



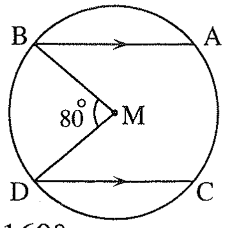
ABCD is a cyclic quadrilateral , m (∠ D) = x° , (∠ B) = 2 x°

, then x = ...

(a) 120° (b) 100°

(c) 60° (d) 50°

(6) **In the opposite figure :**



In a circle M , $\overline{AB} \parallel \overline{CD}$

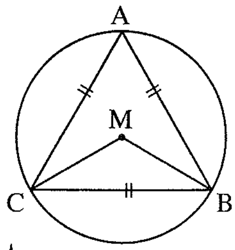
, m ($\angle$ BMD) = 80°

, then m ($\overset{\frown}{AC}$) = ...

(a) 20° (b) 40° (c) 80° (d) 160°

---

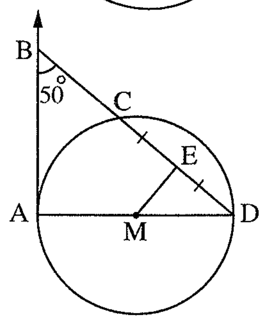
2 [a] **In the opposite figure :**



ABC is an equilateral triangle drawn inside a circle M

Find : (1) m ($\angle$ BAC) (2) m ($\angle$ BMC)

[b] **In the opposite figure :**



$\overline{AD}$ is a diameter of the circle M

, $\overrightarrow{AB}$ is a tangent touches it at A

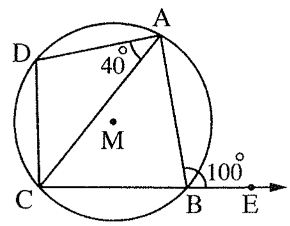
, m ($\angle$ ABC) = 50°

, E is the midpoint of $\overline{DC}$

Find with proof : m ($\angle$ AME)

---

3 [a] **In the opposite figure :**

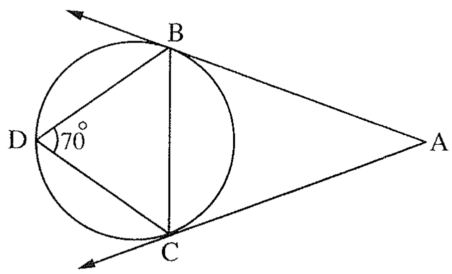


m ($\angle$ ABE) = 100°

, m ($\angle$ CAD) = 40°

Prove that : ADC is an isosceles triangle.

[b] **In the opposite figure :**



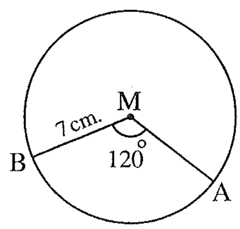
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m ($\angle$ D) = 70°

Find : (1) m ($\angle$ ABC) (2) m ($\angle$ A)

---

4 [a] **In the opposite figure :**

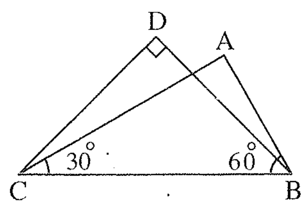


M is a circle with radius length 7 cm.

, m ($\angle$ AMB) = 120°

Find : The length of ($\overset{\frown}{AB}$) $\left(\pi = \frac{22}{7}\right)$

[b] **In the opposite figure :**



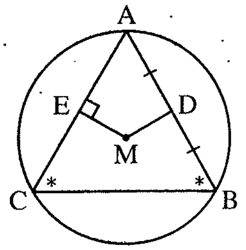
m ($\angle$ BDC) = 90° , m ($\angle$ ACB) = 30°

, m ($\angle$ ABC) = 60°

Prove that :

The points A , B , C and D have one circle passing through them.

5 **[a] In the opposite figure :**



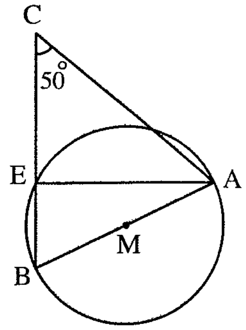
Triangle ABC is inscribed in the circle M , in which

m ($\angle$ B) = m ($\angle$ C) , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$

Prove that : MD = ME

[b] In the opposite figure :



$\overline{AB}$ is a diameter of the circle M

, m ($\angle$ C) = 50°

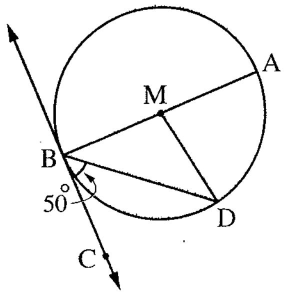
Find with proof : m ($\angle$ CAE)

10 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) **In the opposite figure :**



If m ($\angle$ CBD) = 50°

, then m ($\angle$ AMD) = ...

(a) 40° (b) 50°

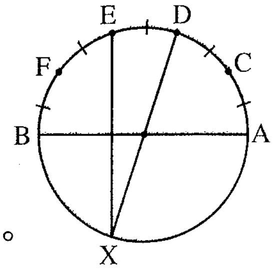
(c) 80° (d) 100°

(2) A circle with diameter length ($2X + 5$) cm. , the straight line L is distant from its centre by ($X + 2$) cm. where $X > 0$, then the straight line is ....

(a) a secant to the circle at two points. (b) lying outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(3) **In the opposite figure :**



If $\overline{AB}$ is a diameter in circle

, m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{EF}$) = m ($\overset{\frown}{FB}$)

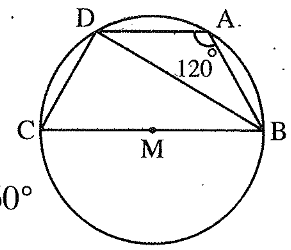
, then m ($\angle$ DXE) = ...

(a) 72° (b) 54° (c) 36° (d) 18°

(4) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm. , then MN $\in$ ....

(a) [3 , 7[(b)]3 , 7[(c)]3 , 7] (d) [3 , 7]

(5) **In the opposite figure :**



If m (∠ BAD) = 120°

, then m (∠ CBD) = ...

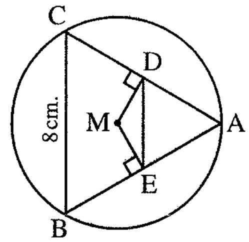
(a) 15° (b) 30° (c) 45° (d) 60°

(6) The number of all common tangents drawn to two distant circles equals ....

(a) 4 (b) 3 (c) 2 (d) 1

---

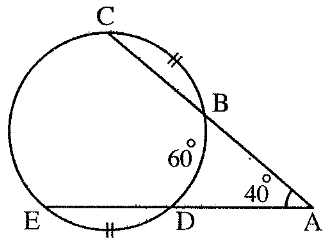
2 **[a] Using the given data in the opposite figure :**



(1) **Prove that :** $\overline{DE} \parallel \overline{CB}$

(2) **Find :** DE

[b] In the opposite figure :



m (∠ A) = 40° , m ($\overset{\frown}{BD}$) = 60°

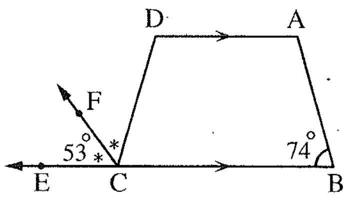
and m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DE}$)

Find with proof :

m ($\overset{\frown}{EC}$) and m ($\overset{\frown}{BC}$)

---

3 **[a] Using the given data in the opposite figure :**



Prove that :

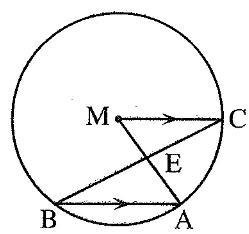
ABCD is a cyclic quadrilateral.

[b] ABCD a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circumcircle of the triangle ABC

---

4 **[a] In the opposite figure :**

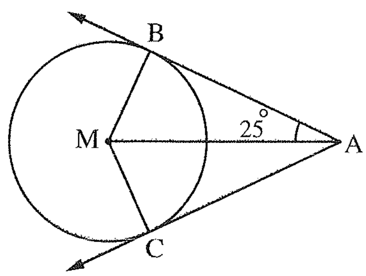


$\overline{AB}$ is a chord in the circle M

, $\overline{CM} \parallel \overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

touch it at B and C respectively and m (∠ BAM) = 25°

(1) **Prove that :** $\overrightarrow{MA}$ bisects (∠ BMC)

(2) **Find :** m (∠ BMC) .

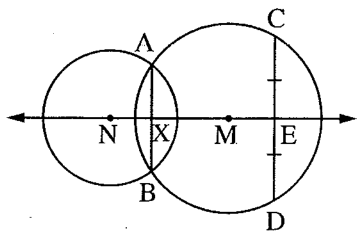
5 **[a] In the opposite figure :**

The two circles M and N intersect at A and B

, $\overline{CD}$ is a chord in the circle M cuts $\overleftrightarrow{MN}$ at E

, if E is the midpoint of $\overline{CD}$

Prove that : $\overline{AB} \parallel \overline{CD}$



[b] ABCD is a square , $\overrightarrow{AX}$ bisects $\angle$ BAC and intersects $\overrightarrow{BD}$ at X

and $\overrightarrow{DY}$ bisects $\angle$ CDB and intersects $\overline{AC}$ at Y

Prove that : AXYD is a cyclic quadrilateral.

11 Damietta Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from the given ones :**

(1) ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and x cm. , then x = ... cm.

(a) 5 (b) 8 (c) 10 (d) 12

(2) If the two circles M , N are touching internally , the length of one radius of them is 3 cm. , MN = 8 cm. , then the length of the radius of the other circle is .... cm.

(a) 5 (b) 11 (c) 6 (d) 12

(3) If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is ....

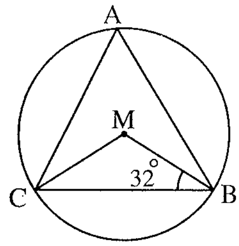
(a) 40° (b) 90° (c) 45° (d) 80°

(4) **In the opposite figure :**

M is a circle , m ($\angle$ MBC) = 32°

, then m ($\overset{\frown}{BC}$ the minor) = ...

(a) 116° (b) 23° (c) 58° (d) 64°



(5) A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2.

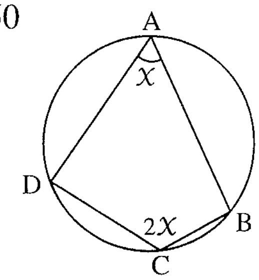
(a) 3050 (b) 3500 (c) 2925 (d) 3250

(6) **In the opposite figure :**

m ($\angle$ A) = x° , m ($\angle$ C) = 2 x°

, then x = ...

(a) 60° (b) 50° (c) 80° (d) 20°



2 [a] A , B are two points where AB = 6 cm. , draw a circle of radius length 5 cm. and passes through the two points A , B

Find : (1) The number of circles can be drawn.

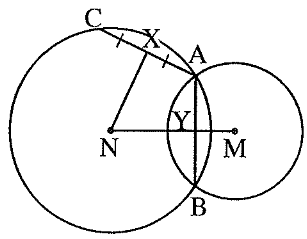
(2) The distance from the centre to $\overline{AB}$ by proof.

[b] In the opposite figure :

M , N are two intersecting circles at A , B , $\overleftrightarrow{MN} \cap \overline{AB} = \{Y\}$

, AB = AC , if X is the midpoint of $\overline{AC}$

Prove that : NX = NY



---

3 [a] $\overline{AB}$, $\overline{AC}$ are two chords in a circle

If X and Y are the two midpoints of $\overset{\frown}{AB}$, $\overset{\frown}{AC}$ respectively , $\overline{XY}$ cuts $\overline{AB}$ at D , $\overline{AC}$ at H

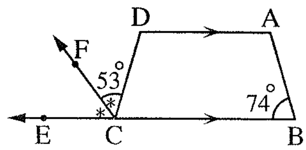
Prove that : AD = AH

[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, m ($\angle$ B) = 74° , m ($\angle$ DCF) = 53°

, $\overrightarrow{CF}$ bisects $\angle$ DCE

Prove that : ABCD is a cyclic quadrilateral.



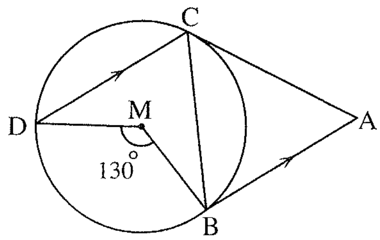
---

4 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB} \parallel \overline{CD}$, m ($\angle$ BMD) = 130°

Prove that : $\overrightarrow{CB}$ bisects $\angle$ ACD



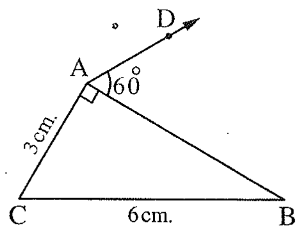
[b] In the opposite figure :

m ($\angle$ BAC) = 90° , m ($\angle$ DAB) = 60°

AC = 3 cm. , BC = 6 cm.

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



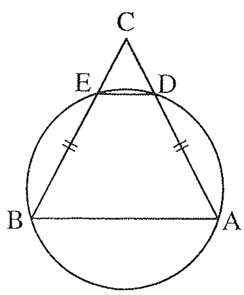
---

5 [a] **In the opposite figure :**

$\overline{AD}$ and $\overline{BE}$ are two equal chords in length in the circle

, $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : CD = CE



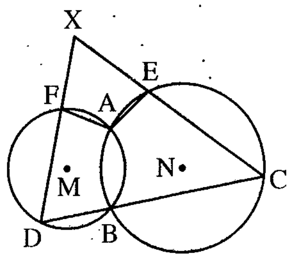
[b] In the opposite figure :

Two intersecting circles at A and B

, $\overline{CD}$ passes through the point B and intersects

the two circles at C and D

Prove that : AFXE is a cyclic quadrilateral.



12 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

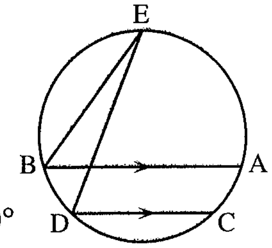
1 **[a] Choose the correct answer from those given :**

(1) **In the opposite figure :**

If m $(\overset{\frown}{AC}) = 30°$, $\overline{AB} \parallel \overline{CD}$

, then m (∠ BED) = ...

(a) 10° (b) 15° (c) 30° (d) 60°



(2) The two tangents drawn from the two ends of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(3) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm.

, then MN ∈ ....

(a)]3 , 7[(b) [3 , 7[(c)]3 , 7] (d) [3 , 7]

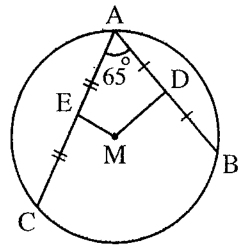
[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two chords in the circle M ,

D , E are the two midpoints of $\overline{AB}$, $\overline{AC}$ respectively

and m (∠ BAC) = 65°

Find : m (∠ DME)



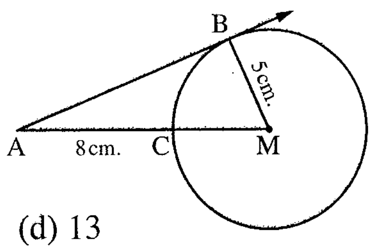
2 **[a] Choose the correct answer from those given :**

(1) **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M

, if MB = 5 cm. , AC = 8 cm. , then AB = ... cm.

(a) 5 (b) 10 (c) 12 (d) 13



(2) The centre of the circumcircle of any triangle is the point of intersection of ....

(a) the interior bisectors of its angles. (b) the exterior bisectors of its angles.

(c) its heights. (d) the symmetric axes of its sides.

(3) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

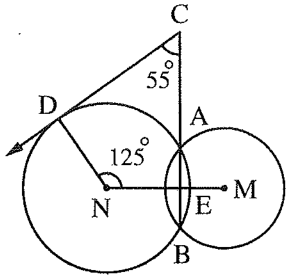
[b] In the opposite figure :

M and N are two intersecting circles at A and B

, C $\in \overrightarrow{BA}$, D $\in$ the circle N

, m ($\angle$ MND) = 125° and m ($\angle$ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



---

3 **[a]** State three cases of the cyclic quadrilateral.

[b] ABCD is a quadrilateral in which AB = AD , m ($\angle$ ABD) = 30° and m ($\angle$ C) = 60°

Prove that : ABCD is a cyclic quadrilateral.

---

4 **[a] Prove that :** The two tangent-segments drawn to a circle from a point outside it are equal in length.

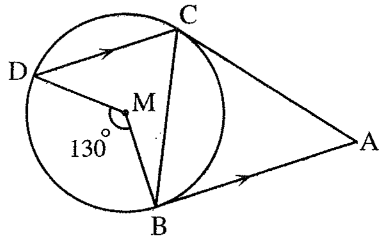
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m ($\angle$ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects $\angle$ ACD

(2) **Find :** m ($\angle$ A) with proof.

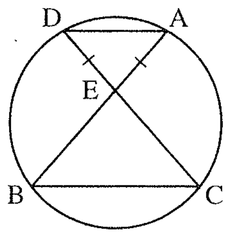


---

5 **[a] In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$, EA = ED

Prove that : EB = EC



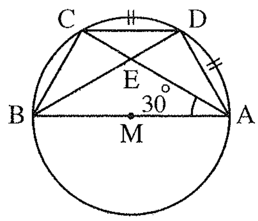
[b] In the opposite figure :

$\overline{AB}$ is a diameter of a circle M , C $\in$ the circle

, m ($\angle$ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{E\}$

(1) **Find :** m ($\angle$ BDC) , m ($\angle$ ABD) with proof.

(2) **Prove that :** Δ ABE is an isosceles triangle.



13 El-Beheira Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer from those given :**

(1) The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units.

(a) – 10 (b) 10 (c) 2 (d) 24

(2) If the length of a diameter of a circle is 7 cm. , and the straight line L at a distance of 3.5 cm. from its centre , then L is ....

(a) a secant to the circle at two points. (b) lying outside the circle.
(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(3) If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)

(4) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

(5) It is possible to draw a circle passing through the vertices of a ....

(a) trapezium. (b) rhombus. (c) parallelogram. (d) rectangle.

(6) The number of tangents can be drawn from a point lies on a circle equals ....

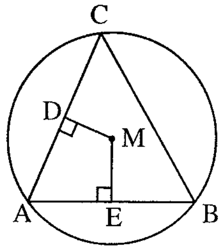
(a) one. (b) two. (c) four. (d) infinite number.

---

2 [a] **In the opposite figure :**

ABC is a triangle drawn inside a circle of centre M
, $\overline{MD} \perp \overline{AC}$, $\overline{ME} \perp \overline{AB}$
, BC = 8 cm.

(1) **Prove that :** $\overline{DE} \,//\, \overline{CB}$ (2) **Find :** DE

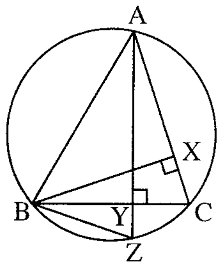


[b] **In the opposite figure :**

ABC is a triangle drawn inside a circle , $\overline{BX} \perp \overline{AC}$
$\overrightarrow{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

(1) ABYX is a cyclie quadrilateral.
(2) $\overline{BC}$ bisects $\angle$ XBZ

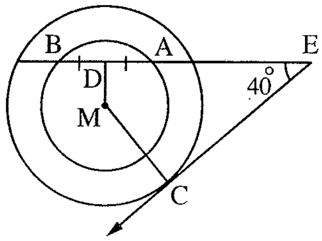


---

3 [a] **In the opposite figure :**

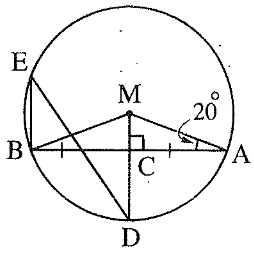
Two concentric circles of centre M
, $\overrightarrow{EC}$ is a tangent to the greater circle
, $\overline{EB}$ cuts the smaller circle at A , B
, D is the midpoint of $\overline{AB}$ and m ($\angle$ CED) = 40°

Find with proof : m ($\angle$ DMC)



[b] $\overline{AB}$, $\overline{CD}$ are two parallel chords in a circle M , E is the midpoint of $\overline{AB}$, $\overleftrightarrow{EM}$ is drawn to cut $\overline{CD}$ at F **Prove that :** FC = FD

4 **[a] In the opposite figure :**



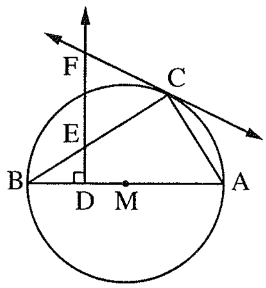
$\overrightarrow{MC} \cap \overline{AB} = \{C\}$, $\overrightarrow{MC} \perp \overline{AB}$

, $\overrightarrow{MC}$ intersects the circle at D

, m ($\angle$ MAB) = 20°

Find : (1) m ($\overset{\frown}{AD}$) (2) m ($\angle$ DEB)

[b] In the opposite figure :



$\overline{AB}$ is a diameter of a circle M

, $\overleftrightarrow{CF}$ is a tangent of the circle at C and $\overrightarrow{DE} \perp \overline{AB}$

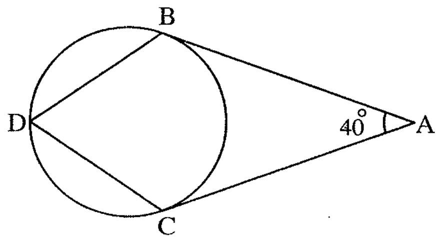
Prove that : (1) ADEC is a cyclic quadrilateral.

(2) FE = FC

---

5 **[a]** Find the measure of the arc which represents $\frac{1}{3}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. $\left(\pi = \frac{22}{7}\right)$

[b] In the opposite figure :



$\overline{AB}$, $\overline{AC}$ are two tangents to the circle at B , C

and m ($\angle$ A) = 40°

Find with proof : m ($\angle$ D)

14 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

(1) If the straight line L is a tangent to the circle of diameter 8 cm. , then the distance between L and the centre equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

(2) The angle whose measure is 50° complements an angle of measure ....

(a) 90° (b) 130° (c) 50° (d) 40°

(3) The inscribed angle which is opposite to the minor arc in a circle is ....

(a) reflex. (b) obtuse. (c) right. (d) acute.

(4) ABC is a triangle in which AB = AC , m ($\angle$ C) = 40° , then m ($\angle$ A) = ...

(a) 40° (b) 80° (c) 100° (d) 120°

(5) The number of the symmetry axes of square is ....

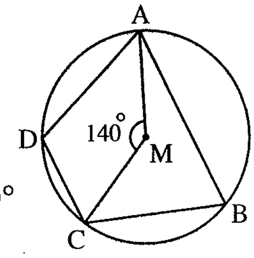
(a) 1 (b) 2 (c) 3 (d) 4

(6) **In the opposite figure :**

In the circle M , if m ($\angle$ AMC) = 140°

, then m ($\angle$ ADC) = ……………

(a) 40° (b) 70° (c) 110° (d) 140°



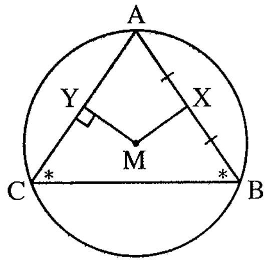
2 **[a] In the opposite figure :**

Triangle ABC is inscribed in circle M , in which :

m ($\angle$ B) = m ($\angle$ C) , X is the midpoint of $\overline{AB}$

, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

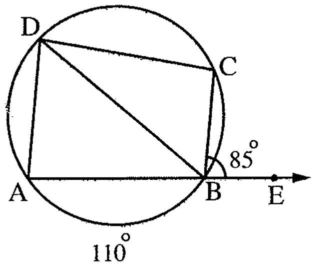


[b] In the opposite figure :

E $\in \overrightarrow{AB}$, E $\notin \overline{AB}$, m ($\widehat{AB}$) = 110°

, m ($\angle$ CBE) = 85°

Find : m ($\angle$ BDC)

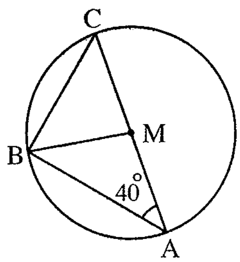


3 **[a] In the opposite figure :**

$\overline{AC}$ is a diameter in a circle M , B $\in$ the circle M

, m ($\angle$ BAC) = 40°

Find : m ($\angle$ CBM)

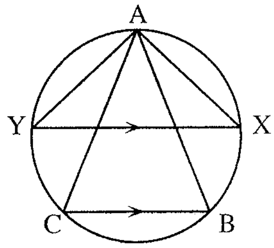


[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

, $\overline{XY}$ // $\overline{BC}$

Prove that : m ($\angle$ XAC) = m ($\angle$ BAY) .



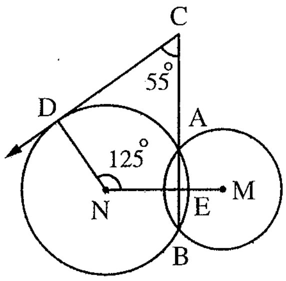
4 **[a] In the opposite figure :**

M and N are two intersecting circles at A and B , C $\in \overrightarrow{BA}$

, D $\in$ the circle N and m ($\angle$ MND) = 125°

, m ($\angle$ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D

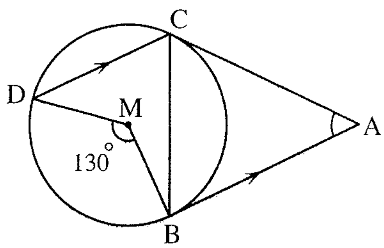


[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$, m ($\angle$ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects $\angle$ ACD (2) **Find :** m ($\angle$ A)



5 **[a] In the opposite figure :**

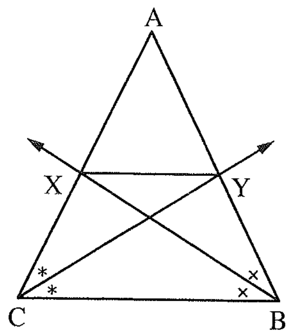
ABC is a triangle in which AB = AC

, $\overrightarrow{BX}$ bisects $\angle$ B and intersect $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects $\angle$ C and intersect $\overline{AB}$ at Y

Prove that : BCXY is a cyclic quadrilateral

and prove that : $\overline{XY}$ // $\overline{BC}$



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY}$ // $\overline{BC}$ **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

15 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

(1) It is impossible to draw a circle passing through the vertices of ....

(a) a triangle. (b) a square. (c) a rhombus. (d) a rectangle.

(2) If m_1 and m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$

(3) M and N are two circles touching internally , their radii lengths are 3 cm. , and 5 cm. , then MN = ... cm.

(a) 2 (b) 3 (c) 5 (d) 8

(4) The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base.

(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3

(5) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

(6) The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm. equals .... cm^2.

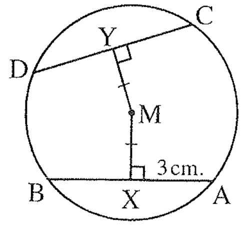
(a) 2 (b) 18 (c) 40 (d) 80

2 **[a] In the opposite figure :**

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, MX = MY

and AX = 3 cm.

Find : The length of $\overline{CD}$



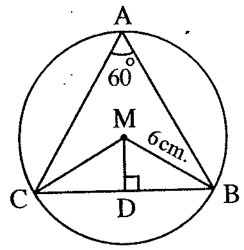
[b] Two concentric circles M , $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C , D , draw $\overline{ME} \perp \overline{AB}$ **Prove that :** AC = BD

3 **[a] In the opposite figure :**

In the circle M , m ($\angle$ A) = 60°

, $\overline{MD} \perp \overline{BC}$, MB = 6 cm.

Find with proof : The length of $\overline{MD}$

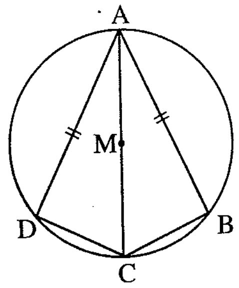


[b] In the opposite figure :

$\overline{AC}$ is a diameter in the circle M

, AB = AD

Prove that : m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{CD})$

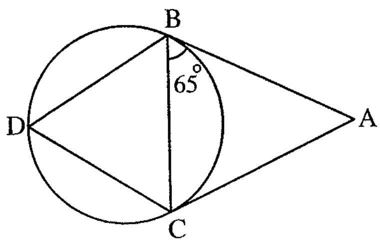


4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m ($\angle$ ABC) = 65°

Find with proof : m ($\angle$ A) and m ($\angle$ D)

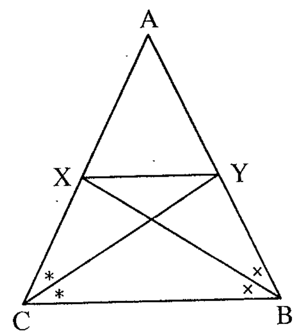


[b] In the opposite figure :

ABC is a triangle in which AB = AC , $\overrightarrow{BX}$ bisects $\angle$ B and intersects $\overline{AC}$ at X

, $\overrightarrow{CY}$ bisects $\angle$ C and intersects $\overline{AB}$ at Y

Prove that : The figure BCXY is a cyclic quadrilateral.

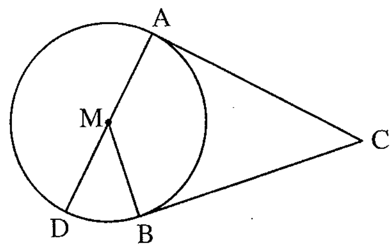


5 **[a] In the opposite figure :**

$\overline{AD}$ is a diameter in a circle of centre M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle at A , B

Prove that : m ($\angle$ DMB) = m ($\angle$ ACB)



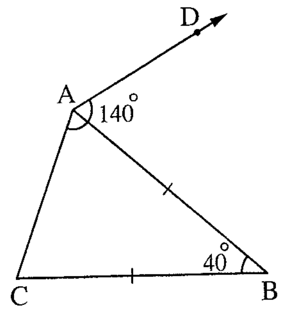
[b] In the opposite figure :

BA = BC , m ($\angle$ DAC) = 140°

and m ($\angle$ B) = 40°

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABC



16 El-Menia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

(1) The two angles A and C in the right-angled triangle at B are ....

(a) complementary. (b) supplementary.

(c) adjacent. (d) vertically opposite angles.

(2) The length of the opposite to the angle of measure 30° in the right-angled triangle is .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

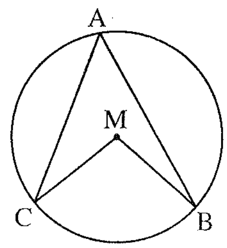
(3) The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is .... cm^2

(a) 2 (b) 14 (c) 24 (d) 48

(4) The number of circles passing through three non-collinear points is ....

(a) 1 (b) zero (c) 2 (d) 3

(5) **In the opposite figure :**



In the circle M ,

if m (∠ M) – m (∠ A) = 50°

, then m (∠ A) = ...

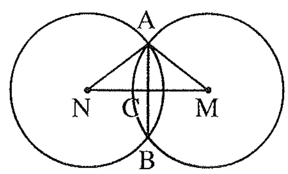
(a) 40° (b) 50° (c) 100° (d) 130°

(6) Which of the following shapes is a cyclic quadrilateral ?

(a) rhombus (b) rectangle (c) parallelogram (d) trapezium

---

2 [a] **In the opposite figure :**

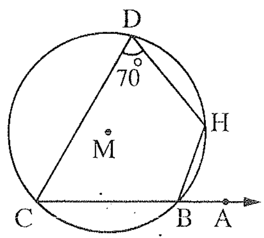


Two congruent circles M and N are intersecting at A and B

If MA = 10 cm. , AB = 12 cm.

Find by proof : The length of $\overline{MN}$

[b] **In the opposite figure :**



BCDH is a cyclic quadrilateral in the circle M

, m (∠ D) = 70° , A $\in \overrightarrow{CB}$, m (∠ C) = $\frac{1}{2}$ m (∠ H)

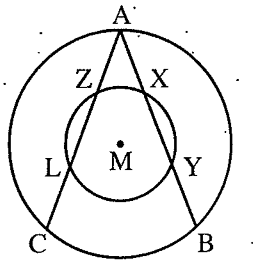
Find by proof : m (∠ ABH) , m (∠ H)

3 **[a] In the opposite figure :**

Two concentric circles at M

, AB = AC

Prove that : XY = ZL

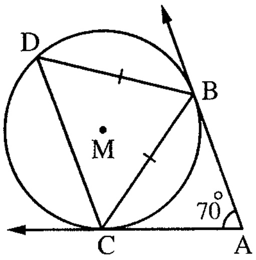


[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M

, m (∠ BAC) = 70° , BC = BD

Find : m (∠ ABD)



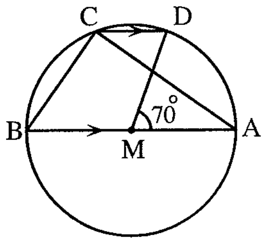
---

4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{DC}$ // $\overline{AB}$, m (∠ AMD) = 70°

Find by proof : m (∠ ACD) , m (∠ ABC)



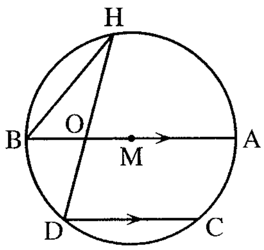
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overline{AB}$ // $\overline{DC}$, m ($\overset{\frown}{DC}$) = 80°

, m ($\overset{\frown}{AH}$) = 100°

Find by proof : m (∠ DHB) , m (∠ AOH)



---

5 **In the opposite figure :**

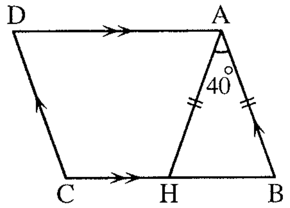
ABCD is a parallelogram

, H ∈ $\overline{BC}$ such that AB = AH , m (∠ BAH) = 40°

(1) **Find :** m (∠ AHB) , m (∠ D)

(2) **Prove that :** AHCD is a cyclic quadrilateral.

(3) **Prove that :** $\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of Δ ABH



17 Assiut Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

(1) The chord which passes through the centre of the circle is called ....

(a) tangent. (b) diameter. (c) radius. (d) side.

(2) The number of symmetry axes of a square ....

(a) 2 (b) 3 (c) 4 (d) 5

(3) The inscribed angle which is opposite to the minor arc in a circle is ....

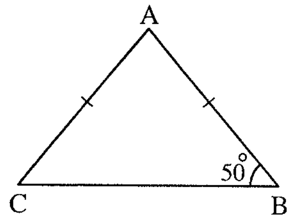
(a) reflex. (b) right. (c) obtuse. (d) acute.

(4) **In the opposite figure :**

ABC is a triangle , AB = AC

, m (∠ B) = 50°

, then m (∠ A) = ...



(a) 100° (b) 90° (c) 80° (d) 70°

(5) A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre.

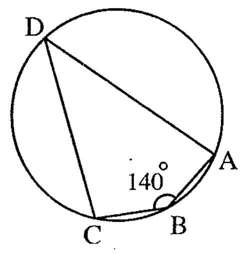
(a) 4 (b) 3 (c) 8 (d) 6

(6) **In the opposite figure :**

m (∠ B) = 140°

, then m (∠ D) = ...

(a) 40° (b) 60° (c) 30° (d) 50°



---

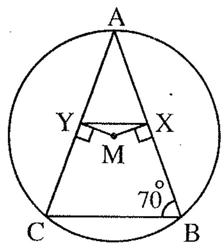
2 [a] **In the opposite figure :**

A circle M , $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{AC}$, m (∠ B) = 70°

(1) **Prove that :** $\overline{XY} \parallel \overline{BC}$

(2) **Find with proof :** m (∠ YXM)



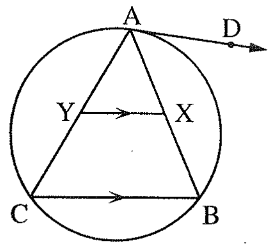
[b] **In the opposite figure :**

$\overline{XY} \parallel \overline{CB}$,

$\overrightarrow{AD}$ is a tangent to the circle at A

Prove that :

$\overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



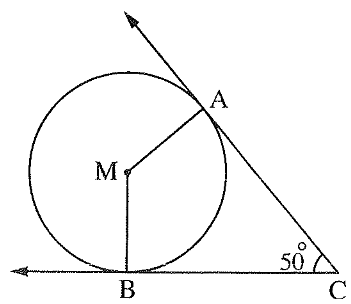
---

3 [a] **In the opposite figure :**

$\overrightarrow{CA}$, $\overrightarrow{CB}$ are two tangents to the circle M

, m (∠ C) = 50°

Find with proof : m (∠ AMB)



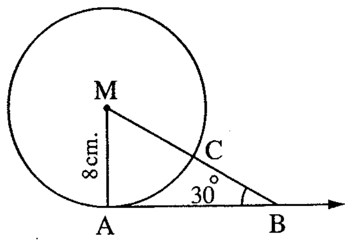
[b] In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M at A and MA = 8 cm.

, m ($\angle$ ABM) = 30°

Find : (1) The length of $\overline{MB}$

(2) m ($\overset{\frown}{CA}$)



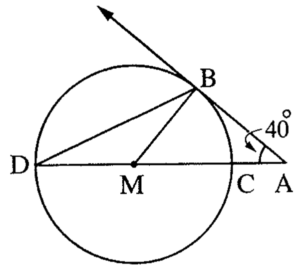
---

4 **[a] In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle at B , m ($\angle$ A) = 40°

, $\overrightarrow{AM}$ intersects the circle M at C and D

Find with proof : m ($\angle$ BDC)



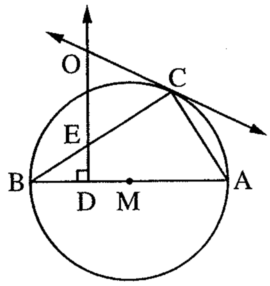
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{CO}$ is a tangent to the circle at C and $\overrightarrow{DO} \perp \overline{AB}$

Prove that : (1) ADEC is a cyclic quadrilateral.

(2) OE = OC



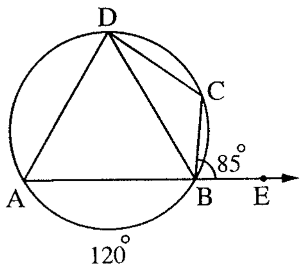
---

5 **[a] In the opposite figure :**

E $\in \overrightarrow{AB}$, E $\notin \overline{AB}$

, m ($\overset{\frown}{AB}$) = 120° , m ($\angle$ CBE) = 85°

Find : m ($\angle$ BDC)



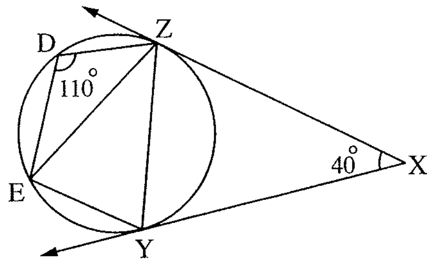
[b] In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle

from the point X , m ($\angle$ X) = 40°

, m ($\angle$ D) = 110°

Prove that : m ($\overset{\frown}{ZE}$) = m ($\overset{\frown}{ZY}$)



18 Souhag Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

(1) The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(2) The number of the axes of symmetry in the equilateral triangle = ...

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(3) M and N are two intersecting circles, their radii lengths are 5 cm. , 2 cm. , then MN ∈ ....

(a) [3 , 7] (b) [3 , 7[(c)]3 , 7] (d)]3 , 7[

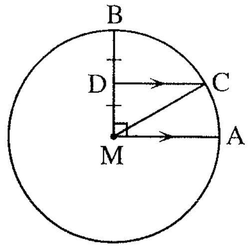
(4) The number of common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(6) **In the opposite figure :**



$\overline{AM}$ // $\overline{CD}$, MD = DB

, m (∠ AMB) = 90° , then m $(\overset{\frown}{AC})$ = ...

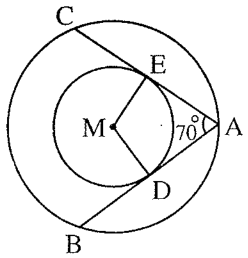
(a) 45° (b) 60°

(c) 30° (d) 90°

---

2 [a] Find the measure of the arc which represents $\frac{1}{2}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. $\left(\pi = \frac{22}{7}\right)$

[b] In the opposite figure :



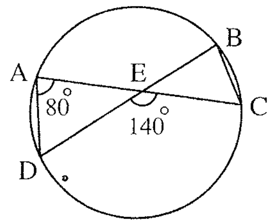
Two concentric circle at M , $\overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle at D and E , m (∠ A) = 70°

(1) **Find :** m (∠ DME)

(2) **Prove that :** AB = AC

---

3 [a] **In the opposite figure :**

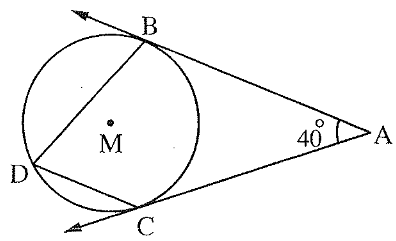


m (∠ CED) = 140°

, m (∠ A) = 80°

Find : m (∠ C)

[b] In the opposite figure :



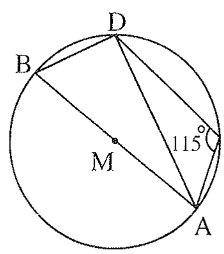
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 40°

Find with proof : m (∠ D)

---

4 [a] **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M ,

m (∠ ACD) = 115°

Find with proof : m (∠ DAB)

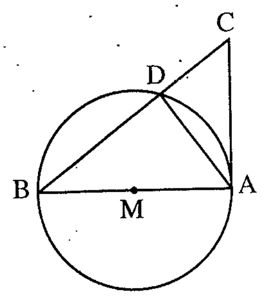
[b] In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, $\overline{AC}$ is a tangent touches it at A

, if AC = 9 cm. and BM = 6 cm.

Find : The lengths of $\overline{BC}$ and $\overline{AD}$



5 **[a]** State three cases of cyclic quadrilateral.

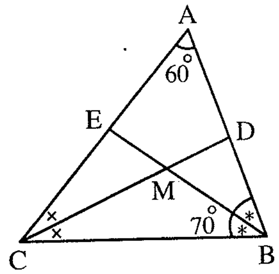
[b] In the opposite figure :

m (∠ A) = 60° , $\overrightarrow{BE}$ bisects ∠ ABC

, m (∠ B) = 70° , $\overrightarrow{CD}$ bisects ∠ ACB

(1) **Find :** m (∠ BMC)

(2) **Prove that :** ADME is a cyclic quadrilateral.



19 Qena Governorate

Answer the following questions : ***(Calculators are Permitted)***

1 **Choose the correct answer :**

(1) If the area of the circle M = 16 π cm^2 , A is a point on its plane where MA = 8 cm. , then A is ....

(a) outside the circle. (b) inside the circle.

(c) on the circle. (d) on the centre of the circle.

(2) A tangent to a circle of diameter length 6 cm. is at distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

(3) The centre of the circumcircle of the triangle is the intersection point of its ....

(a) altitudes of triangle. (b) medians of a triangle.

(c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.

(4) The inscribed angle drawn in a semicircle is .... angle.

(a) acute. (b) obtuse. (c) right. (d) straight.

(5) The two tangent-segments drawn from a point outside a circle are ....

(a) equal in length. (b) not equal in length.

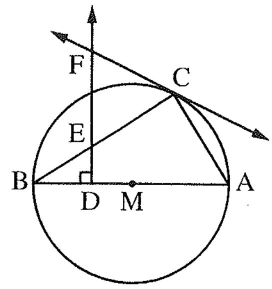
(c) perpendicular. (d) parallel.

(6) The figure is said to be cyclic quadrilateral if the measure of any exterior angle at any vertex equal to .... of the interior angle at the opposite vertex.

(a) the measure. (b) half the measure.

(c) twice the measure. (d) third the measure.

2 **[a] In the opposite figure :**



$\overline{AB}$ is a dimeter in the circle M

, $\overleftrightarrow{CF}$ is a tangent to the circle at C , $\overrightarrow{DE} \perp \overline{AB}$

Prove that :

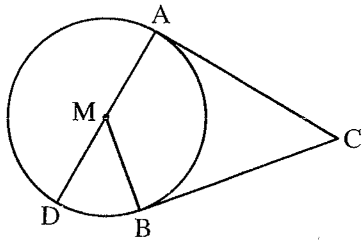
(1) ADEC is a cyclic quadrilateral.

(2) FE = FC

[b] The length of $\overline{AB}$ is 4 cm. , draw a circle of radius length 3 cm. and passes through the two points A , B how many circles can be drawn ? Find the radius length of the smallest circle that can be drawn to pass through the two points A , B

---

3 **[a] In the opposite figure :**

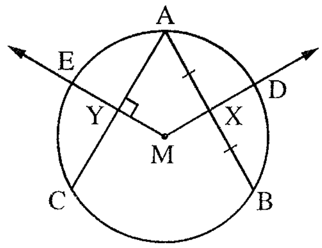


$\overline{AD}$ is a diameter in the circle M

, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are two tangents to the circle M at A and B respectively

Prove that : m ($\angle$ DMB) = m ($\angle$ ACB)

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in circle M

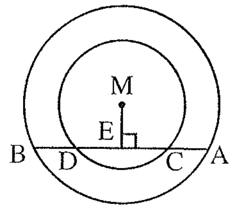
and X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D

, $\overrightarrow{MY} \perp \overline{AC}$ intersects it at Y and intersects the circle at E

Prove that : XD = YE

---

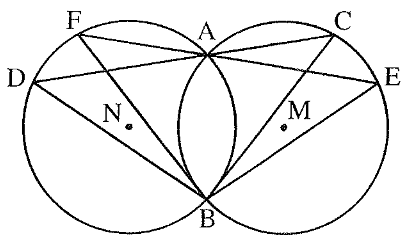
4 **[a] In the opposite figure :**



Two concentric circles M

, $\overline{AB}$ is a chord in the larger circle intersecting the smaller circle at C and D , $\overline{ME} \perp \overline{AB}$ **Prove that :** AC = BD

[b] In the opposite figure :



M and N are two intersecting circles at A and B

, $\overleftrightarrow{AC}$ intersects the circle M at C

and intersects the circle N at D ,

$\overleftrightarrow{AE}$ intersects the circle M at E

and intersects the circle N to F

Prove that : m ($\angle$ EBC) = m ($\angle$ FBD)

---

5 ABC is an acute-angled triangle drawn inside a circle , draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and cuts the circle at E , then draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N

Porve that : (1) ANDC is a cyclic quadrilateral. (2) m ($\angle$ BND) = m ($\angle$ BED)

20 Luxor Governorate

محافظة الأقصر

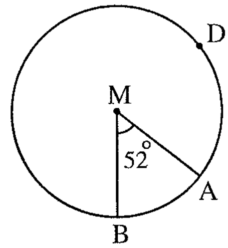
Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the accumulative angles at a point = ... °

(a) 80 (b) 120 (c) 360 (d) 630

(2) **In the opposite figure :**



If m (∠ AMB) = 52°

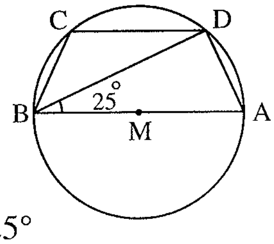
, then m $(\widehat{ADB})$ = ... °

(a) 52 (b) 104 (c) 128 (d) 308

(3) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the hypotenuse length.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

(4) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

, then m (∠ C) = ...

(a) 50° (b) 100° (c) 115° (d) 125°

(5) The sum of lengths of any two sides of a triangle .... the length of the third side.

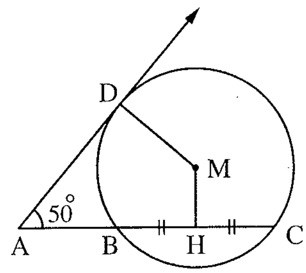
(a) < (b) > (c) = (d) ≤

(6) The number of circles pass by three non-collinear points = ...

(a) infinite number. (b) 3 (c) 1 (d) 0

---

2 [a] **In the opposite figure :**



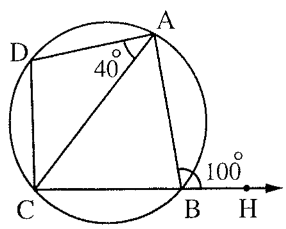
$\overrightarrow{AD}$ is a tangent to the circle at D ,

H is the midpoint of $\overline{BC}$

, m (∠ A) = 50°

Find with proof : m (∠ DMH)

[b] **In the opposite figure :**



m (∠ ABH) = 100°

, m (∠ DAC) = 40°

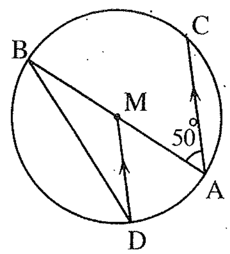
Prove that : m $(\widehat{CD})$ = m $(\widehat{AD})$

3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ // $\overline{MD}$, m (∠ CAB) = 50°

Find : m (∠ MDB)



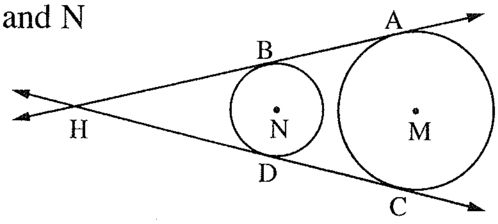
[b] In the opposite figure :

$\overleftrightarrow{AH}$ and $\overleftrightarrow{CH}$ are two tangents to the two circles M and N

touch the circle M at A and C

touch the circle N at B and D

Prove that : AB = CD

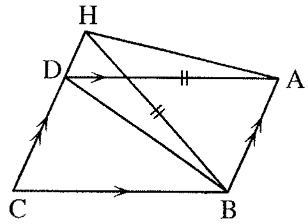


4 **[a] In the opposite figure :**

ABCD is a parallelogram H ∈ $\overrightarrow{CD}$

where BH = AD

prove that : ABDH is a cyclic quadrilateral.



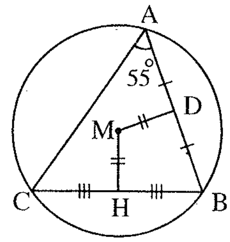
[b] In the opposite figure :

D is the midpoint of $\overline{AB}$

, H is the midpoint of $\overline{BC}$,

m (∠ A) = 55° , MD = MH

Find : m (∠ B)



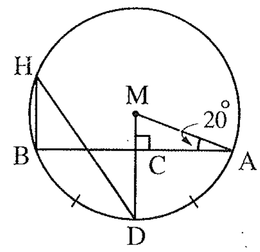
5 **[a] In the opposite figure :**

$\overrightarrow{MC} \perp \overline{AB}$ and intersects the circle M at D

which is the midpoint of $\overset{\frown}{AB}$

, m (∠ MAB) = 20°

Find : (1) m ($\overset{\frown}{AD}$) (2) m (∠ DHB)

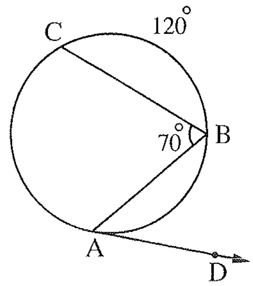


[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle at A

, m (∠ B) = 70° , m ($\overset{\frown}{BC}$) = 120°

Find : m (∠ BAD)

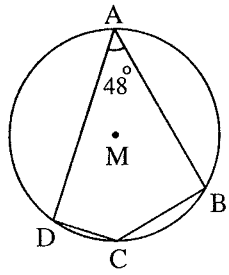


21 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from the given ones :

(1) In the opposite figure :

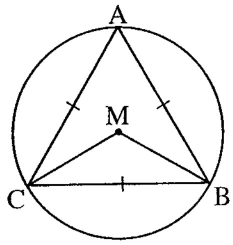


m (∠ A) = 48° , then

the measure of major arc $\overset{\frown}{BD}$ = ...

(a) 260° (b) 265° (c) 264° (d) 262°

(2) In the opposite figure :

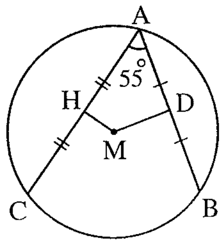


ABC is an equilateral triangle inscribed in circle M

, then m (∠ BMC) = ...

(a) 50° (b) 120° (c) 60° (d) 100°

(3) In the opposite figure :



D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

, m (∠ A) = 55°

, then m (∠ DMH) = ...

(a) 120° (b) 130° (c) 135° (d) 125°

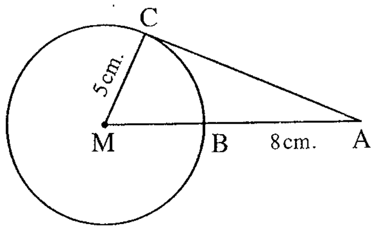
(4) Number of axes of symmetry of the circle = ...

(a) zero (b) one (c) infinite number. (d) 4

(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) 2

(6) In the opposite figure :



$\overline{AC}$ is a tangent to circle M at C

if MC = 5 cm. , AB = 8 cm.

, then AC = ... cm.

(a) 5 (b) 10 (c) 13 (d) 12

---

2 [a] M and N are two circles of radii length 9 cm. and 4 cm. respectively.

Show the position of each of them with respect to the other if :

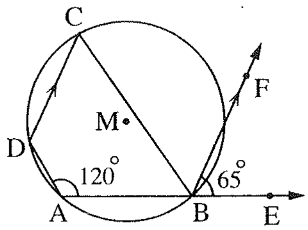
(1) MN = 5 cm. (2) MN = 10 cm.

[b] In the opposite figure :

ABCD is a quadrilateral inscribed in circle M

, $\overrightarrow{BF} \parallel \overline{DC}$, m (∠ EBF) = 65° , m (∠ BAD) = 120°

Find : m (∠ ADC)



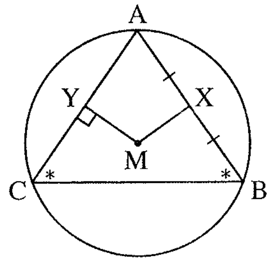
---

3 **[a] In the opposite figure :**

ABC is a triangle inscribed in circle M ,

m (∠ B) = m (∠ C) , X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

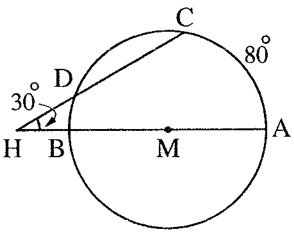


[b] In the opposite figure :

$\overline{AB}$ is a diameter in circle M , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$,

m (∠ AHC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$



---

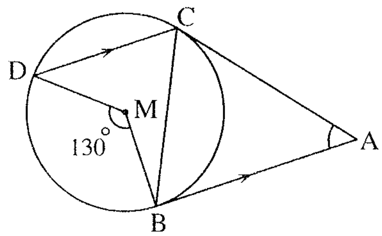
4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

at B and C , $\overline{AB} \parallel \overline{CD}$, m (∠ BMD) = 130°

(1) **Find :** m (∠ ABC)

(2) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

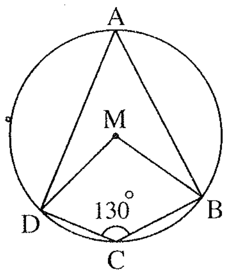


[b] In the opposite figure :

In the circle M ,

if m (∠ BCD) = 130°

Find : m (∠ BMD)



---

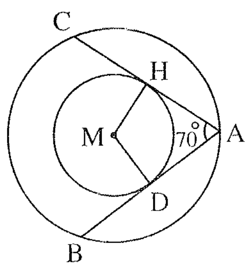
5 **[a] In the opposite figure :**

Two concentric circles at M

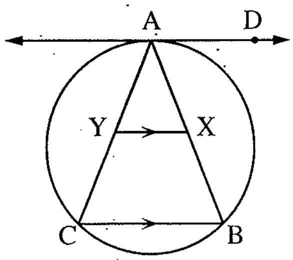
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to smaller circle at D and H

, m (∠ BAC) = 70°

Prove that : (1) AB = AC (2) **Find :** m (∠ DMH)



[b] In the opposite figure :



ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{AD}$ is a tangent to a circle at A

, X $\in \overline{AB}$, Y $\in \overline{AC}$, $\overline{XY}$ // $\overline{BC}$

Prove that :

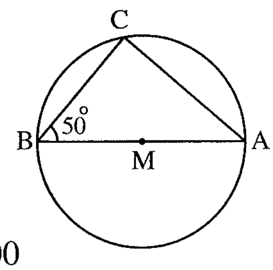
$\overleftrightarrow{AD}$ is a tangent to the circle which passes through the points A , X , Y

22 South Sinai Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

m ($\angle$ ABC) = 50° , then m $(\overset{\frown}{BC})$ = ...°

(a) 40 (b) 50 (c) 80 (d) 100

(2) The rhombus in which the lengths of diagonals are 6 cm. and 8 cm. its area = ... cm.^{2}

(a) 12 (b) 14 (c) 24 (d) 48

(3) If M is a circle of radius length r cm. , then the length of the simicircle = ... cm.

(a) $2 \pi r$ (b) $\frac{1}{4} \pi r$ (c) $\frac{1}{2} \pi r$ (d) πr

(4) The longest chord in the circle is called ....

(a) diameter. (b) tangent. (c) secant. (d) radius.

(5) The image of the point (2 , 3) by rotation R (O , 180°) is the point ....

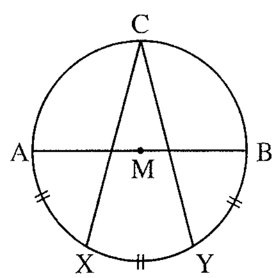
(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

(6) The sum of measures of the two opposite angles in the cyclic quadrilateral equal ....°

(a) 180 (b) 120 (c) 100 (d) 30

---

2 [a] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M

, the length of $(\overset{\frown}{AX})$ = the length of $(\overset{\frown}{XY})$ = the length of $(\overset{\frown}{BY})$

find with proof : m ($\angle$ C)

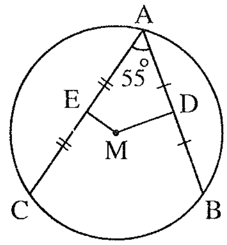
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$,

m ($\angle$ BAC) = 55°

Find with proof : m ($\angle$ DME)

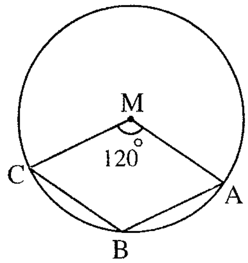


---

3 **[a] In the opposite figure :**

M is a circle and m ($\angle$ AMC) = 120°

Find with proof : m ($\angle$ ABC)



[b] Two circles M and N with radii lengths of 7 cm. and 4 cm. respectively

Show the position of each of them respect to the other in the following cases :

(1) MN = 8 cm. (2) MN = 3 cm. (3) MN = 12 cm.

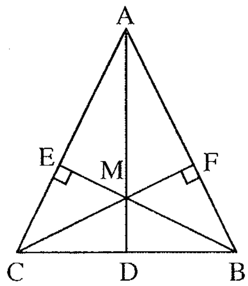
---

4 **[a] In the opposite figure :**

Δ ABC , $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$

$\overrightarrow{AM} \cap \overleftrightarrow{BC} = \{D\}$

Prove that : MDCE is a cyclic quadrilateral.

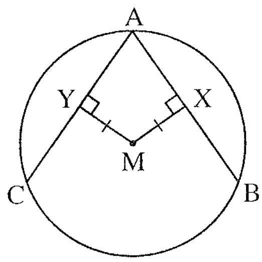


[b] In the opposite figure :

M is a circle , $\overline{AB}$ and $\overline{AC}$ are two chords ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$, AB = 6 cm. , MX = MY

Find with proof : The length of $\overline{AY}$



---

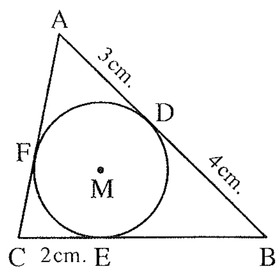
5 **[a] In the opposite figure :**

M is an inscribed circle in the triangle ABC

and touches its sides at D , E and F

, AD = 3 cm. , CE = 2 cm. , BD = 4 cm.

Find with proof : The perimeter of Δ ABC

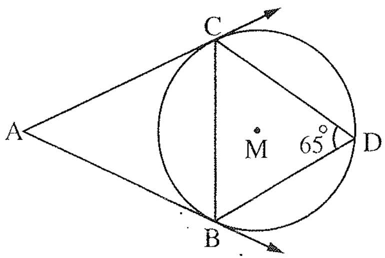


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents of the circle M

, m ($\angle$ D) = 65°

Find with proof : m ($\angle$ A)



23 Red Sea Governorate

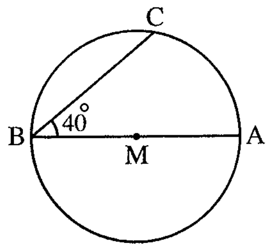
Answer the following questions :

1 Choose the correct answer from the given ones :

(1) Number of the circles that pass through three non-collinear points equals ....

(a) zero (b) one (c) three (d) an infinite number

(2) **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m (∠ ABC) = 40° , then m $(\overset{\frown}{BC})$ = ...

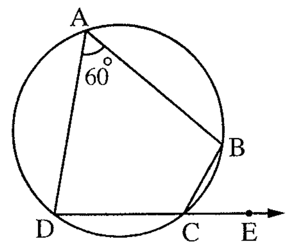
(a) 40° (b) 50°

(c) 90° (d) 100°

(3) If the two circles M and N are touching externally , their radii lengths are 9 cm. , r cm. , and MN = 14 cm. , then r = ... cm.

(a) 5 (b) 7 (c) 10 (d) 23

(4) **In the opposite figure :**

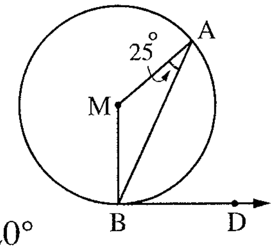


If m (∠ BAD) = 60° , then m (∠ BCE) = ...

(a) 30° (b) 60°

(c) 80° (d) 120°

(5) **In the opposite figure :**



If $\overrightarrow{BD}$ is a tangent to the circle M

, m (∠ BAM) = 25°

, then m (∠ ABD) = ...

(a) 25° (b) 50° (c) 65° (d) 120°

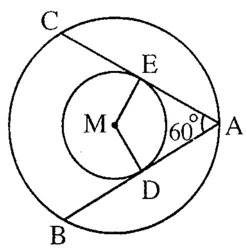
(6) Circumference of a circle is 6 π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is ....

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) the diameter to the circle.

---

2 [a] **In the opposite figure :**



Two concentric circles M ,

$\overline{AB}$, $\overline{AC}$ are two tangents to the smaller circle , m (∠ A) = 60°

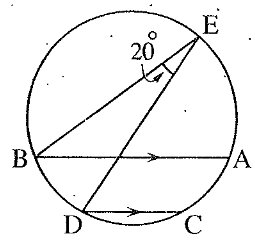
(1) **Find :** m (∠ DME) (2) **Prove that :** AB = AC

[b] In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two parallel chords

, m (∠ BED) = 20°

Find : m ($\overset{\frown}{AC}$)



---

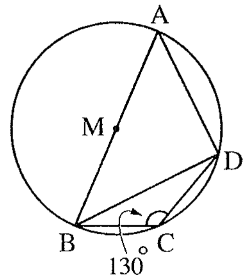
3 **[a] In the opposite figure :**

ABCD is a quadriteral inscribed in a circle M

where M ∈ $\overline{AB}$

, m (∠ BCD) = 130°

Find : m (∠ A) , m (∠ ABD)



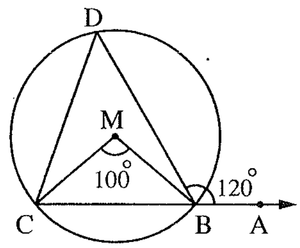
[b] In the opposite figure :

In the circle M :

m (∠ BMC) = 100°

, m (∠ ABD) = 120°

Find with proof : m (∠ DCB)



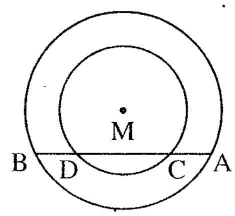
---

4 **[a] In the opposite figure :**

Two concentric circle M

, $\overline{AB}$ is a chord in the large circle intersecting the small circle at C and D

Prove that : AC = BD

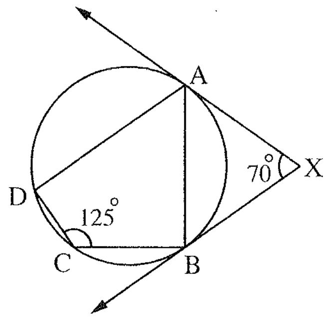


[b] In the opposite figure :

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to a circle at A and B

, m (∠ AXB) = 70° , m (∠ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects ∠ DAX



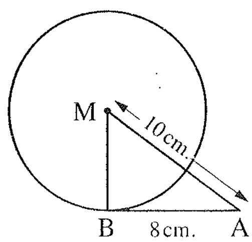
---

5 **[a] In the opposite figure :**

$\overline{AB}$ is a tangent to a circle M at B

, AB = 8 cm. , AM = 10 cm.

Find : The area of Δ ABM



[b] ABC is a triangle inscribed in a circle , $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$ where $\overline{XY}$ // $\overline{BD}$

Prove that : AXYC is a cyclic quadrilateral.

24 Matrouh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer :

(1) The perimeter of the square whose area is 81 cm^2 is ....

(a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.

(2) The two opposite angles in the cyclic quadrilateral are ....

(a) equal. (b) complementary. (c) supplementary. (d) alternate.

(3) ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 40° , then m ($\angle$ A) = ...

(a) 40° (b) 50° (c) 90° (d) 130°

(4) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 240°

(5) The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2

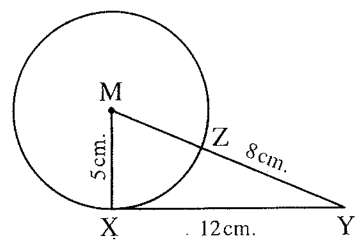
(a) 6 (b) 10 (c) 30 (d) 60

(6) If the two circles M , N are touching internally , the radius length of one of them is 3 cm. , and MN = 8 cm. , then the radius length of the other circle equals ....

(a) 5 cm. (b) 6 cm. (c) 11 cm. (d) 12 cm.

---

2 [a] In the opposite figure :



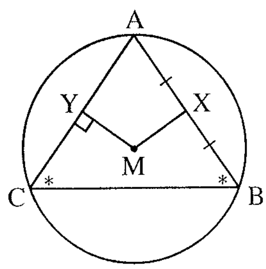
M is a circle whose radius length is 5 cm.

, XY = 12 cm , $\overline{MY} \cap$ the circle M = {Z}

and ZY = 8 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X

[b] In the opposite figure :



Δ ABC is inscribed in the circle M

, in which m ($\angle$ B) = m ($\angle$ C)

, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

---

3 [a] Prove that : The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

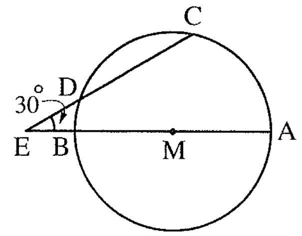
[b] ABCD is a quadrilateral drawn in a circle , $F \in \overline{AB}$

, draw $\overrightarrow{FE} \parallel \overline{CB}$ to cut $\overline{CD}$ at E , $\overrightarrow{DF} \cap \overrightarrow{CB} = \{X\}$

Prove that : (1) AFED is a cyclic quadrilateral. (2) m ($\angle$ BXF) = m ($\angle$ EAD)

---

4 **[a] In the opposite figure :**



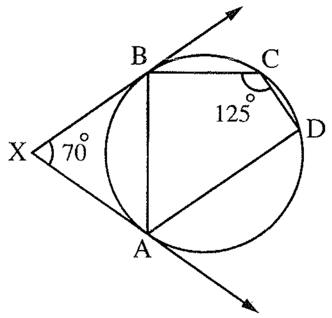
$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m ($\angle$ AEC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$

[b] In the opposite figure :



$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

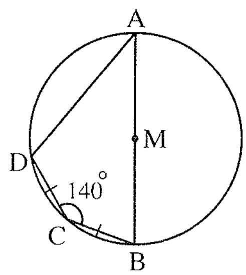
, m ($\angle$ AXB) = 70° , m ($\angle$ DCB) = 125°

Prove that : $\overrightarrow{AB}$ bisects $\angle$ DAX

---

5 **[a]** Mention three cases of the cyclic quadrilateral.

[b] In the opposite figure :



ABCD is a quadrilateral inscribed in the circle M

where $M \in \overline{AB}$, CB = CD

, m ($\angle$ BCD) = 140°

Find : (1) m ($\angle$ A) (2) m ($\angle$ D)